Changes in northern hemisphere temperature variability shaped by regional warming patterns

Article

Supplemental Material

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Supplementary information

1 Table of models

The list of 26 Coupled Model Intercomparison Project 5 (CMIP5) models that were used in this study. The columns denote, from left to right, the model index, the model name, its horizontal resolution (in degrees) and the number of atmospheric vertical levels. In all models, the high emissions representative concentration pathway 8.5 (RCP8.5) scenario is used for the projected runs, where the radiative forcing increases to about 8.5 W m^{-2} by the year of 2100. The historical runs include all the observed atmospheric forcing (including both anthropogenic and natural sources). For all models, the single run of ensemble member r1i1p1 is used. The data was obtained from the World Data Center for Climate (WDCC), available at http://cera-www.dkrz.de/WDCC/ui/.

Model index	Model name	Horizontal resolution (longitude$^\circ$ $\times$ latitude$^\circ$)	Vertical resolution (number of levels)
1	ACCESS1-0	1.9 × 1.2	38
2	ACCESS1-3	1.9 × 1.2	38
3	BCC-CSM1-1*	2.8 × 2.8	26
4	BCC-CSM1-1-M	2.8 × 2.8	26
5	BNU-ESM*	2.8 × 2.8	26
6	CanESM2	2.8 × 2.8	35
7	CCSM4*	1.2 × 0.9	26
8	CMCC-CM*	0.7× 0.7	31
9	CNRM-CM5*	1.4 × 1.4	31
10	CSIRO-Mk3-6-0	1.9 × 1.9	18
11	FGOALS-g2*	2.8 × 3.0	26
12	GFDL-CM3*	2.5 × 2.0	48
13	GFDL-ESM2G*	2.5 × 2.0	24
14	GFDL-ESM2M*	2.5 × 2.0	24
15	HadGEM-CC	1.9 × 1.3	38
16	HadGEM-ES	1.9 × 1.3	60
17	INMCM4	2.0 × 1.5	21
18	IPSL-CM5A-MR*	2.5 × 1.3	39
19	IPSL-CM5A-LR*	3.8 × 1.9	39
20	MIROC5*	1.4 × 1.4	40
21	MIROC-ESM*	2.8 × 2.8	80
22	MIROC-ESM-CHEM*	2.8 × 2.8	80
23	MPI-ESM-LR	1.9 × 1.9	47
24	MPI-ESM-MR	1.9 × 1.9	95
25	MRI-CGCM3*	1.1 × 1.1	48
26	NorESM1-M*	2.5 × 1.9	26

Table S1: List of 26 CMIP5 models included in this study, ordered alphabetically. Shown are the model index, name, horizontal resolution (in degrees) and the number of atmospheric vertical levels. Models denoted by * are those for which the near-surface (T2m) temperature data was used to calculate the surface temperature response (presented in Extended Data Fig. 7 and Fig. 8).

2 Approximate expressions for variance and skewness

Assume that the temperature in a region is characterized by warm anomalies T_w and cold anomalies $-T_c$ (i.e., T_c is the amplitude of the cold anomalies), where for simplicity we take these values to be constants. (If the temperature anomalies are represented by distributions, e.g., sinusoidal, rather than single values, then the same expressions result, but with multiplicative constants in front which depend on the shape of the distribution.) Denote the time length (number of non-consecutive time intervals) with positive and negative departures as τ_w and τ_c for the warm and cold anomalies, respectively, such that $\tau_w + \tau_c = \tau$ is the overall time length of the record. For the averaged temperature anomaly to be zero, $\overline{T'} = 0$, it must be that $T_w \tau_w = T_c \tau_c$. Hence, one finds that $\tau_w = \frac{T_c}{T_w + T_c}\tau$, and $\tau_c = \frac{T_w}{T_w + T_c}\tau$ (such that, for example, if the magnitude of the warm anomalies is larger than the cold anomalies, then the time length of the warm anomalies should be smaller in order for the average to be zero).

It then follows that the variance, denoted as σ^2, is given by

$$\sigma^2 = \frac{(T_c {T_w}^2 + T_w {T_c}^2)}{T_w + T_c} = T_w T_c, \tag{1}$$

while skewness is given by

$$S = \frac{(T_c {T_w}^3 - T_w {T_c}^3)}{T_w + T_c} \left(\sigma^2\right)^{-\frac{3}{2}} = \frac{T_w T_c \left(T_w - T_c\right)}{\sigma^3}. \tag{2}$$

Hence, from (1) and (2), it follows that

$$\sigma S = T_w - T_c. \tag{3}$$

To derive more intuitive forms, (1) can be written as

$$\sigma^2 = T_w T_c = \left(\frac{1}{2}(T_w + T_c)\right)^2 - \left(\frac{1}{2}(T_w - T_c)\right)^2.$$

Assuming that the difference between T_w and T_c is small, the second expression on the RHS is negligible to order $(T_w - T_c)$. Therefore an approximate form of (1) is

$$\sigma^2 \approx \left(\frac{1}{2}(T_w + T_c)\right)^2, \tag{4}$$

and the standard deviation is

$$\sigma \approx \frac{1}{2}(T_w + T_c). \tag{5}$$

Then (3) and (5) give for the skewness

$$S \approx \frac{T_w - T_c}{\frac{1}{2}(T_w + T_c)}. \tag{6}$$

Expressions (4) and (6) are the approximate forms for variance and skewness given in equations (1) and (2) in the manuscript, respectively. The approximation made in deriving (4) (i.e., small asymmetry between warm and cold anomalies) is then seen to be valid for small skewness S, which is a reasonable assumption given the relatively small skewness values (smaller than one) found in this study.

3 Approximated variance and skewness change

Given the expression (1) for variance above, $\sigma^2 = T_w T_c$, we find

$$\Delta\left(\sigma^2\right) = \Delta T_w T_c + \Delta T_c T_w = T_w T_c \left(\frac{\Delta T_w}{T_w} + \frac{\Delta T_c}{T_c}\right),$$

hence

$$\Delta\left(\sigma^2\right) = \sigma^2 \left(\frac{\Delta T_w}{T_w} + \frac{\Delta T_c}{T_c}\right).$$

Using $T_w = \left|\eta_{\mathrm{S}} \overline{T}_{S_y}\right|$, $T_c = \left|\eta_{\mathrm{N}} \overline{T}_{N_y}\right|$, and $\Delta T_w = |\eta_{\mathrm{S}}| \Delta \left|\overline{T}_{S_y}\right|$, $\Delta T_c = |\eta_{\mathrm{S}}| \Delta \left|\overline{T}_{N_y}\right|$ (where we have assumed for simplicity that η_S and η_N remain the same in the future climate; see section 6 above), gives

$$\Delta\left(\sigma^2\right) \approx \sigma^2 \left(\frac{\Delta\left|\overline{T}_{S_y}\right|}{\left|\overline{T}_{S_y}\right|} + \frac{\Delta\left|\overline{T}_{N_y}\right|}{\left|\overline{T}_{N_y}\right|}\right),$$

which can be rewritten, using $\frac{d|f|}{dx} = sgn(f)\frac{df}{dx}$, as

$$\Delta\left(\sigma^2\right) \approx \sigma^2 \left(\frac{\Delta\overline{T}_{S_y}}{\overline{T}_{S_y}} + \frac{\Delta\overline{T}_{N_y}}{\overline{T}_{N_y}}\right), \tag{7}$$

which is the approximation for the variance change given in equation (3) in the manuscript.

Similarly, from expression (3) above $S = \frac{T_w - T_c}{\sigma}$, hence

$$\Delta S = \frac{\sigma\left(\Delta T_w - \Delta T_c\right) - \Delta\sigma(T_w - T_c)}{\sigma^2}.$$

Substituting $\sigma^2 = T_w T_c$, and using the approximate expression (5) for the standard deviation in the limit of small skewness, $\sigma \approx \frac{1}{2}(T_w + T_c)$ and therefore $\Delta\sigma \approx \frac{1}{2}(\Delta T_w + \Delta T_c)$, we find

$$\Delta S \approx \frac{\frac{1}{2}(T_w + T_c)\left(\Delta T_w - \Delta T_c\right) - \frac{1}{2}\left(\Delta T_w + \Delta T_c\right)\left(T_w - T_c\right)}{T_w T_c} = \frac{T_c \Delta T_w - T_w \Delta T_c}{T_w T_c},$$

hence

$$\Delta S \approx \frac{\Delta T_w}{T_w} - \frac{\Delta T_c}{T_c},$$

or

$$\Delta S \approx \frac{\Delta\overline{T}_{S_y}}{\overline{T}_{S_y}} - \frac{\Delta\overline{T}_{N_y}}{\overline{T}_{N_y}}, \tag{8}$$

which is the approximation for the skewness change given in equation (4) in the manuscript.

4 Approximate magnitudes of warm and cold temperature anomalies

Given the approximations for variance and skewness above in the limit of small asymmetry, $\sigma^2 \approx \left(\frac{1}{2}(T_w + T_c)\right)^2$ and $S \approx \frac{T_w - T_c}{\frac{1}{2}(T_w + T_c)}$, respectively, we can estimate the intensities of warm and cold temperature anomalies as follows.

From the variance expression, one finds

$$T_w + T_c \approx 2\sigma, \tag{9}$$

and from the skewness equation,

$$T_w - T_c \approx S\sigma. \tag{10}$$

Hence, with no further approximations,

$$T_c \approx \sigma\left(1 - \frac{S}{2}\right), \tag{11}$$

and

$$T_w \approx \sigma\left(1 + \frac{S}{2}\right), \tag{12}$$

which recover expressions (5) and (6) in the manuscript for the cold and warm temperature anomalies, respectively.