Horses for courses: mean-variance for asset allocation and 1/N for stock selection

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Online Appendices for

Horses for Courses: Mean-Variance for Asset Allocation and 1/N for Stock Selection

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A1 S&P Sectors and the Largest Constituents

The details of the S&P sectors and stocks are summarised in Table A1.1.

Sector	Constituents
SP4EFIN	JPM, BAC, WFC, C (Citi Group), MS, AXP(American Express), PNC, AIG, MMC(Marsh & McLennan Cos.), ALL(Allstate Corp.)
SP5ETEL	T(AT&T Inc.), VZ(Verizon Communications Inc.), CTL(CenturyLink Inc.)
SP5EHCR	JNJ(Johnson & Johnson), PFE(Pfizer Inc.), UNH(UnitedHealth Group Inc.), MRK(Merck & Co. Inc.), AMGN(Amgen Inc.), BMY(Bristol-Myers Squibb Co.), ABT(Abbott Laboratories), LLY(Eli Lilly & Co.), AET(Aetna Inc.), BAX(Baxter International Inc.)
SP5ECST	WMT, PG(Procter & Gamble Co.), KO(Coca-Cola Co.), PEP(PepsiCo Inc.), MO(Altria Group Inc.), CL(Colgate-Palmolive Co.), KMB(Kimberly-Clark Corp.), GIS(General Mills Inc.), K(Kellogg Co.), CPB(Campbell Soup Co.)
SP5EIND	GE, BA(Boeing Co.), MMM(3M), HON(Honeywell International Inc.), UNP(Union Pacific Corp.), CAT(Caterpillar Inc.), RTN(Raytheon Co.), CSX(CSX Group), EMR(Emerson Electric Co.), NSC(Norfolk Southern Corp.)
SP5EMAT	PX(Praxair Inc.), SHW(Sherwin-Williams Co.), APD(Air Products & Chemicals Inc.), PPG(PPG Industries Inc.), IP(International Paper Co.), NEM(Newmont Mining Corp.), NUE(Nucor Corp.), FMC(FMC Corp.), EMN(Eastman Chemical Co.), IFF(International Flavors & Fragrances Inc.)
SP5ECOD	HD(Home Depot Inc.), DIS(Walt Disney Co.), MCD(McDonald's Corp.), F(Ford Motor Co.), CCL(Carnival Corp.), TGT(Target Corp.), GPC(Genuine Parts Co.), LB(L Brands Inc.), GPS(Gap Inc.), GT(Goodyear Tire & Rubber Co.)
SP5EUTL	NEE(NextEra Energy Inc.), DUK(Duke Energy Corp.), D(Dominion Energy Inc.), SO(Southern Co.), AEP(American Electric Power Co. Inc.), EXC(Exelon Corp.), PCG(PG&E Corp.), ED(Consolidated Edison Inc.), EIX(Edison International), PEG(Public Service Enterprise Group Inc.)
SP5EENE	XOM(Exxon Mobil Corp.), CVX(Chevron Corp.), SLB(Schlumberger Ltd.), COP(ConocoPhillips), EOG(EOG Resources Inc.), OXY(Occidental Petroleum Corp.), HAL(Halliburton Co.), BHGE(Baker Hughes, a GE co.), HES(Hess Corp.), MRO(Marathon Oil Corp.)
SP5EINT	MSFT, ORCL(Oracle Corp.), INTC(Intel Corp.), CSCO(Cisco Systems Inc.), IBM, ADP(Automatic Data Processing Inc.), HPQ(HP Inc.), CA(CA Inc.), MSI(Motorola Solutions Inc.), XRX(Xerox Corp.)

Table A1.1: S&P Sectors and the Largest Constituents

A2 Performance Metrics

The Sharpe ratio (Sharpe, 1966) is one of the most popular metrics for measuring risk-adjusted returns, and is defined as:

$$SR = \frac{\overline{\mu}_P - \overline{r}_f}{\sigma_P},$$

where $\overline{\mu}_P - \overline{r}_f$ represents the out-of-sample mean excess portfolio return, and σ_P is the portfolio standard deviation over the entire out-of-sample (investment) period. The Sharpe ratio has its limitations, and for this reason we also employ several other metrics.

The Certainty Equivalent Return (CERs) for mean-variance investors can be approximated and computed as follows:

$$CER = \overline{\mu}_P - \frac{\lambda \sigma_P^2}{2},$$

where λ is the relative risk aversion parameter, and $\overline{\mu}_P$ and σ_P have been defined above.

The Omega ratio (Keating and Shadwick, 2002) with a target return of zero, also known as the average gain to the average loss ratio, is computed as follows:

$$Omega = \frac{\sum_{t=1}^{\tau} \max(0, R_{p,t})}{\sum_{t=1}^{T} \max(0, -R_{p,t})}$$

where τ is the sample size of the out of sample observations, $R_{p,t}$ denotes the out-of-sample portfolio return at time t. The main advantage of the Omega ratio is that it does not require any assumption about the distribution of portfolio returns.

The Dowd ratio is the out-of-sample mean excess portfolio return, divided by the portfolio value-at-risk (Prigent, 2007), and is computed as follows:

$$Dowd = \frac{\overline{\mu}_P - \overline{r}_f}{VaR_{95\%}}.$$

The VaR in the Dowd ratio has been computed at the 95% confidence level over the entire investment (out-of-sample) period.

A3 Significance Tests for Sharpe Ratio and CER

A3.1 For Comparing Sharpe Ratios

Given two portfolios k and n, with $\hat{\mu}_k$, $\hat{\mu}_n$, $\hat{\sigma}_k^2$, $\hat{\sigma}_n^2$, and $\hat{\sigma}_{k,n}$ as their estimated means, variances, and covariance over a sample of size τ, the test of the hypothesis H_0:

$$\frac{\hat{\mu}_k}{\hat{\sigma}_k} = \frac{\hat{\mu}_n}{\hat{\sigma}_n}$$

is obtained by the test statistic Z_{JK}, which is asymptotically distributed as a standard normal:

$$Z_{\text{JK}} = \frac{\hat{\sigma}_n \hat{\mu}_k - \hat{\sigma}_k \hat{\mu}_n}{\sqrt{\vartheta}}$$

where

$$\vartheta = \frac{2\hat{\sigma}_k^2 \hat{\sigma}_n^2 - 2\hat{\sigma}_k \hat{\sigma}_n \hat{\sigma}_{k,n} + \frac{\hat{\sigma}_n^2 \hat{\mu}_k^2}{2} - \frac{\hat{\mu}_n \hat{\mu}_k \hat{\sigma}_{k,n}^2}{\hat{\sigma}_k \hat{\sigma}_n}}{\tau}.$$

A3.2 For Comparing CERs

If ν denotes the vector of moments, $\nu = (\mu_k, \mu_n, \sigma_k^2, \sigma_n^2)$, then $\hat{\nu}$, its empirical counterpart, is obtained from a sample of size τ. The difference between the CERs of the two strategies k and n is,

$$f(\nu) = \left(\mu_k - \frac{\lambda \sigma_k^2}{2}\right) - \left(\mu_n - \frac{\lambda \sigma_n^2}{2}\right)$$

and the asymptotic distribution of $f(\nu)$ is

$$\sqrt{T}\left[f(\hat{\nu}) - f(\nu)\right] \rightarrow N\left(0, \frac{\partial f^{\intercal}}{\partial \nu} \Xi \frac{\partial f}{\partial \nu}\right)$$

where

$$\Xi = \begin{bmatrix} \sigma_k^2 & \sigma_{k,n} & 0 & 0 \\ \sigma_{k,n} & \sigma_n^2 & 0 & 0 \\ 0 & 0 & 2\sigma_k^4 & 2\sigma_{k,n}^2 \\ 0 & 0 & 2\sigma_{k,n}^2 & 2\sigma_n^4 \end{bmatrix}.$$

A4 Proof of Proposition 1

For Λ, we derive $\beta^\intercal \left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)^{-1} \beta$ and $\beta^\intercal \frac{\mathbb{1}\mathbb{1}^\intercal}{\mathbb{1}^\intercal\left(\sigma_b^2\beta\beta^\intercal+\sigma_\varepsilon^2 I\right)\mathbb{1}}\beta$ separately:

$$\beta^\intercal \left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)^{-1} \beta = \frac{\beta^\intercal \left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)^\dagger \beta}{\det\left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)} = \frac{B_1}{B_1\sigma_b^2 + \sigma_\varepsilon^2};$$

$$\beta^\intercal \frac{\mathbb{1}\mathbb{1}^\intercal}{\mathbb{1}^\intercal\left(\sigma_b^2\beta\beta^\intercal+\sigma_\varepsilon^2 I\right)\mathbb{1}}\beta = \frac{\mathbb{1}^\intercal\beta\beta^\intercal\mathbb{1}}{\sigma_b^2\mathbb{1}^\intercal\beta\beta^\intercal\mathbb{1} + N\sigma_\varepsilon^2} = \frac{B_2}{B_2\sigma_b^2 + N\sigma_\varepsilon^2}.$$

For Υ, we derive $\alpha^\intercal \left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)^{-1} \alpha$ and $\alpha^\intercal \frac{\mathbb{1}\mathbb{1}^\intercal}{\mathbb{1}^\intercal\left(\sigma_b^2\beta\beta^\intercal+\sigma_\varepsilon^2 I\right)\mathbb{1}}\alpha$ separately:

$$\begin{aligned}
\alpha^\intercal \left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)^{-1} \alpha &= \frac{\alpha^\intercal \left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)^\dagger \alpha}{\det\left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)} \\
&= \frac{\sigma_\varepsilon^2 \alpha^\intercal \left(\sigma_b^2\left[\left(\beta\beta^\intercal + I\right)^\dagger - I\right] + \sigma_\varepsilon^2 I\right)\alpha}{\det\left(\sigma_b^2 \beta\beta^\intercal + \sigma_\varepsilon^2 I\right)} \\
&= \frac{k\sigma_b^2 C}{(B_1\sigma_b^2 + \sigma_\varepsilon^2)\sigma_\varepsilon^2} + \frac{kA_1}{B_1\sigma_b^2 + \sigma_\varepsilon^2};
\end{aligned}$$

$$\alpha^\intercal \frac{\mathbb{1}\mathbb{1}^\intercal}{\mathbb{1}^\intercal\left(\sigma_b^2\beta\beta^\intercal+\sigma_\varepsilon^2 I\right)\mathbb{1}}\alpha = \frac{\mathbb{1}^\intercal\alpha\alpha^\intercal\mathbb{1}}{\sigma_b^2\mathbb{1}^\intercal\beta\beta^\intercal\mathbb{1} + N\sigma_\varepsilon^2} = \frac{A_2}{B_2\sigma_b^2 + N\sigma_\varepsilon^2}.$$

To prove $\frac{\partial\Lambda}{\partial(\sigma_\varepsilon^2)} > 0$, we can easily compute

$$\begin{aligned}
\frac{\partial\Lambda}{\partial\left(\sigma_\varepsilon^2\right)} &= \mu_b^2\left[N\frac{B_2}{\left(B_2\sigma_b^2 + N\sigma_\varepsilon^2\right)^2} - k\frac{B_1}{\left(B_1\sigma_b^2 + \sigma_\varepsilon^2\right)^2}\right] \\
&> \mu_b^2\left[\frac{\frac{B_2}{N}}{\left(\frac{B_2}{N}\sigma_b^2 + \frac{\sigma_\varepsilon^2}{k}\right)^2} - \frac{\frac{B_1}{k}}{\left(\frac{B_1}{k}\sigma_b^2 + \frac{\sigma_\varepsilon^2}{k}\right)^2}\right].
\end{aligned}$$

The inequation above is by the fact $k < 1$ which is straightforward to verify (see (21) in Kan and Zhou, 2007). Define $f(x)$ as

$$f(x) \triangleq \frac{x}{\left(x\sigma_b^2 + \frac{\sigma_\varepsilon^2}{k}\right)^2},$$

then we have

$$\frac{\partial\Lambda}{\partial\left(\sigma_\varepsilon^2\right)} > \mu_b^2\left[f\left(\frac{B_2}{N}\right) - f\left(\frac{B_1}{k}\right)\right].$$

Since $\frac{\partial f(x)}{\partial x} = \frac{k^2}{\left(xk\sigma_b^2+\sigma_\varepsilon^2\right)^2}\left(\sigma_\varepsilon^2 - xk\sigma_b^2\right)$ and $\frac{B_1}{k} > \frac{B_2}{N}$ (from the condition $B_1 > B_2\frac{k}{N} > \frac{\sigma_\varepsilon^2}{\sigma_b^2}$), $f(x)$ is decreasing in $\frac{B_2}{N} \le x \le \frac{B_1}{k}$ (when $x \ge \frac{B_2}{N}$, $\sigma_\varepsilon^2 - xk\sigma_b^2 < 0$ by the condition $B_1 > B_2\frac{k}{N} > \frac{\sigma_\varepsilon^2}{\sigma_b^2}$), i.e., $f\left(\frac{B_2}{N}\right) > f\left(\frac{B_1}{k}\right) \implies \frac{\partial \Lambda}{\partial(\sigma_\varepsilon^2)} > 0$. This completes the proof of $\frac{\partial \Lambda}{\partial(\sigma_\varepsilon^2)} > 0$.

To prove $\frac{\partial \Upsilon}{\partial(\sigma_\varepsilon^2)} < 0$, we focus on $\frac{kA_1}{B_1\sigma_b^2+\sigma_\varepsilon^2} - \frac{A_2}{B_2\sigma_b^2+N\sigma_\varepsilon^2}$ first:

$$\begin{aligned}\frac{\partial\left(\frac{kA_1}{B_1\sigma_b^2+\sigma_\varepsilon^2} - \frac{A_2}{B_2\sigma_b^2+N\sigma_\varepsilon^2}\right)}{\partial\left(\sigma_\varepsilon^2\right)} &= \frac{NA_2}{\left(B_2\sigma_b^2+N\sigma_\varepsilon^2\right)^2} - \frac{kA_1}{\left(B_1\sigma_b^2+\sigma_\varepsilon^2\right)^2} \\ &= \frac{1}{\left(\frac{B_2}{N}\sqrt{\frac{N}{A_2}}\sigma_b^2 + \sqrt{\frac{N}{A_2}}\sigma_\varepsilon^2\right)^2} - \frac{1}{\left(\frac{B_1}{\sqrt{kA_1}}\sigma_b^2 + \frac{1}{\sqrt{kA_1}}\sigma_\varepsilon^2\right)^2}.\end{aligned}$$

Given the condition $A_1k > A_1k\left(\frac{B_2}{B_1N}\right)^2 > \frac{A_2}{N}$, we can readily verify that

$$\frac{B_2}{N}\sqrt{\frac{N}{A_2}} > \frac{B_1}{\sqrt{kA_1}} \text{ and } \frac{N}{A_2} > \frac{1}{kA_1},$$

meaning

$$\frac{\partial\left(\frac{kA_1}{B_1\sigma_b^2+\sigma_\varepsilon^2} - \frac{A_2}{B_2\sigma_b^2+N\sigma_\varepsilon^2}\right)}{\partial\left(\sigma_\varepsilon^2\right)} < 0.$$

Next, with C being positive due to its quadratic form, it is clear that

$$\frac{\partial\left(\frac{k\sigma_b^2C}{(B_1\sigma_b^2+\sigma_\varepsilon^2)\sigma_\varepsilon^2}\right)}{\partial\left(\sigma_\varepsilon^2\right)} < 0,$$

therefore $\frac{\partial \Upsilon}{\partial(\sigma_\varepsilon^2)} < 0$ is proved.

A5 Robustness Check on Using the Financial Sector to Calibrate the US S&P Simulations

For the simulations in section 7 we calibrated the distribution of alphas using the US finance sector. To test the robustness of using the financial sector, we also ran these simulations using the health care sector, the industrial sector and the consumer discretional sector to calibrate the alpha distribution. The results appear in Figures A5.1 to A5.3; and show that the simulation results in the paper are robust to changing the sector used to calibrate the alpha distribution.

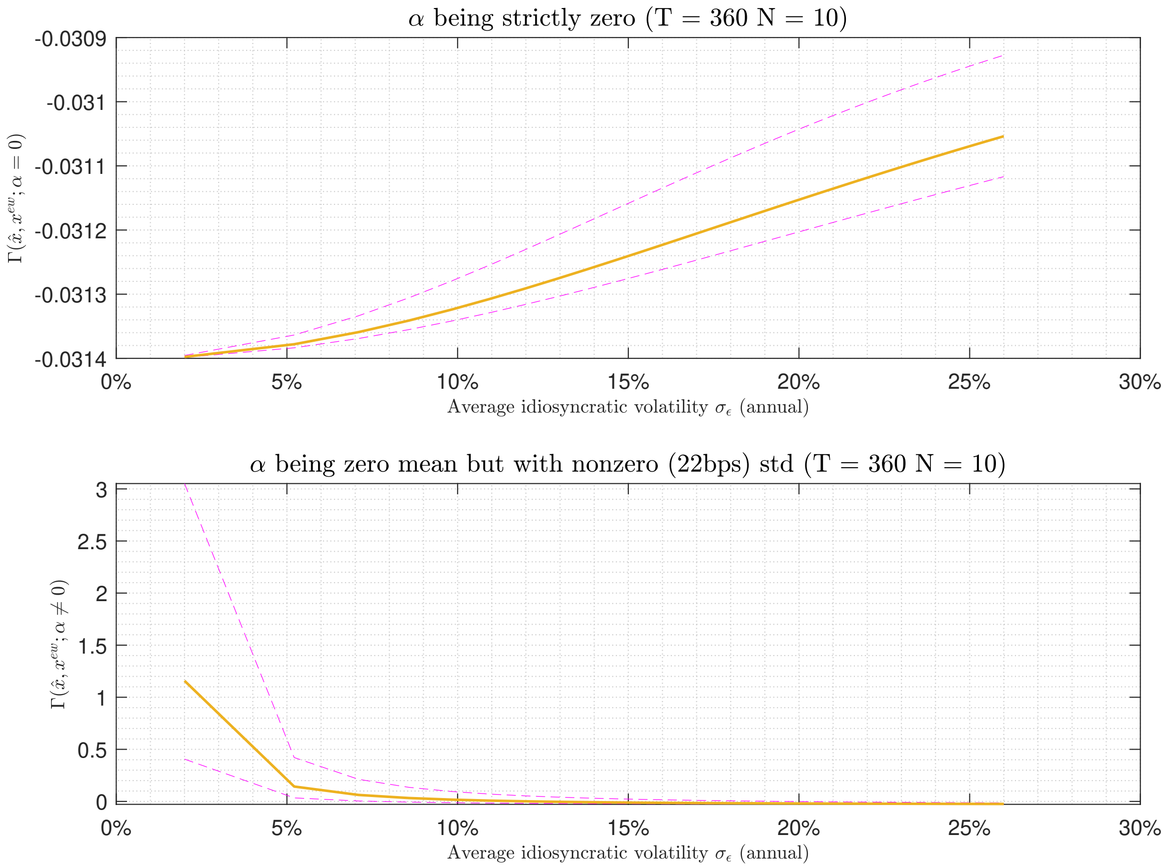

Figure A5.1: $\Gamma\left(\hat{\boldsymbol{x}}, \boldsymbol{x}^{\mathrm{ew}}\right)$ versus Idiosyncratic Volatility (σ_ε). The simulation is based on $T = 240$, $N = 10$ and alpha distribution from the Health Care sector. The dashed lines are 95% confidence intervals

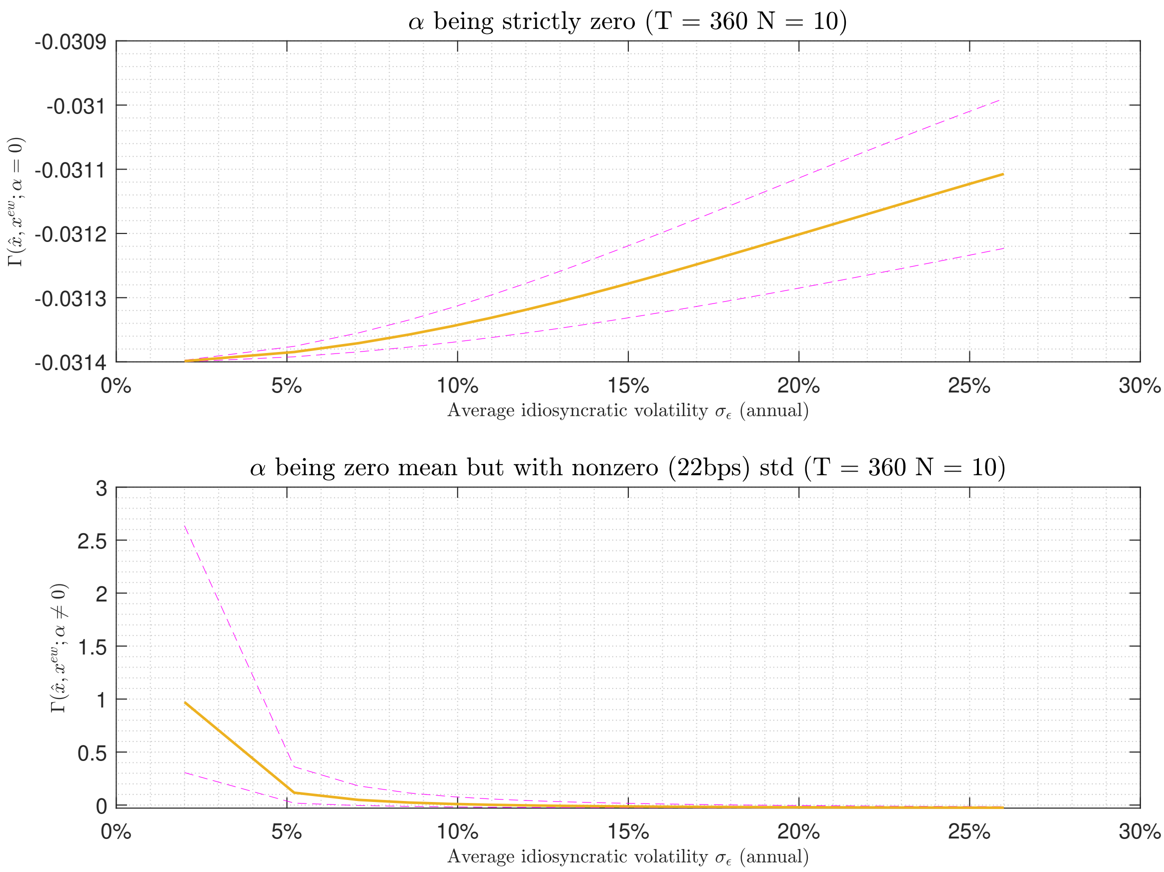

Figure A5.2: $\Gamma\left(\hat{\boldsymbol{x}}, \boldsymbol{x}^{\text{ew}}\right)$ versus Idiosyncratic Volatility (σ_ε). The simulation is based on $T = 240$, $N = 10$ and alpha distribution from the Industrial sector. The dashed lines are 95% confidence intervals

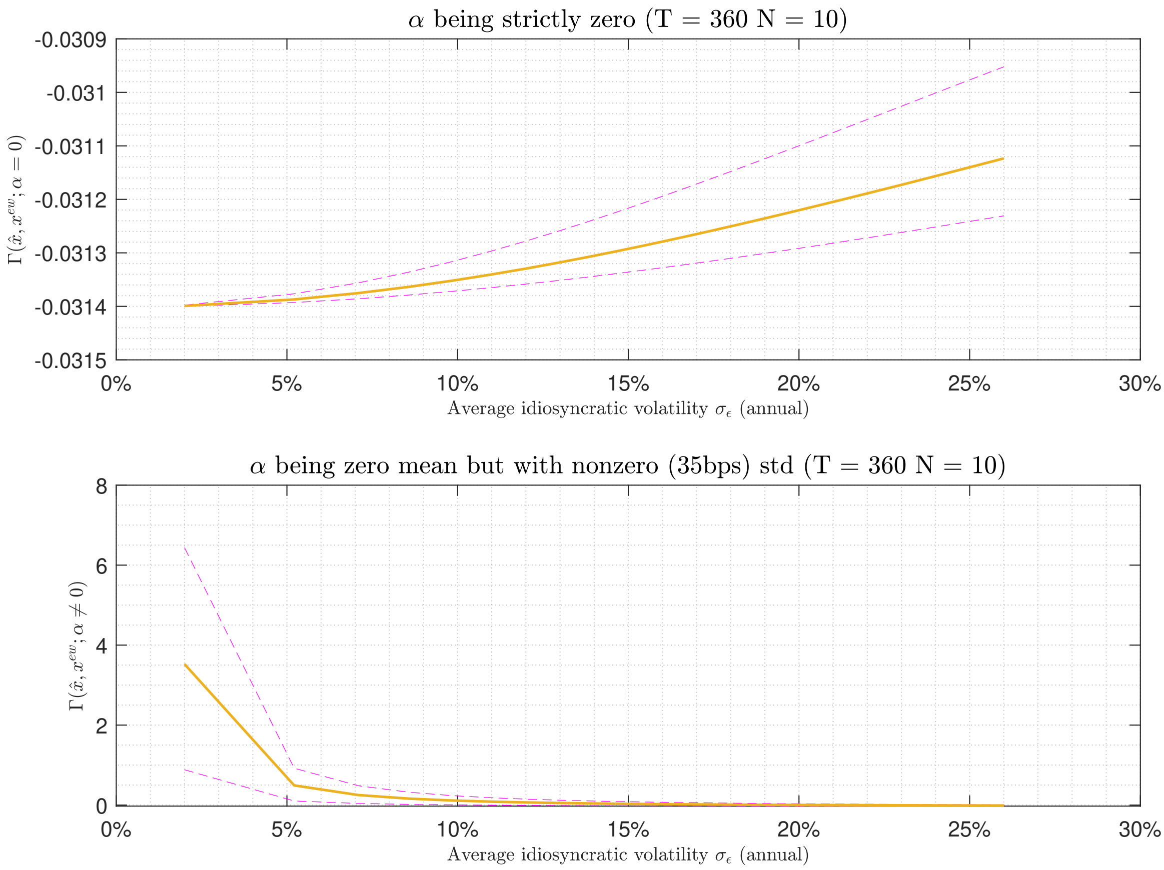

Figure A5.3: $\Gamma\left(\hat{\boldsymbol{x}}, \boldsymbol{x}^{\mathrm{ew}}\right)$ versus Idiosyncratic Volatility (σ_ε). The simulation is based on $T = 240$, $N = 10$ and alpha distribution from the Consumer Discretionary sector. The dashed lines are 95% confidence intervals

A6 Fama-French 5, 10, 12 and 17 Industries

In a robustness check we applied our core analysis to the Fama-French 5, 10, 12 and 17 industry portfolios using monthly data for these companies from January 1994 to August 2017. In each case we analysed the ten largest companies in January 1994 and, allowing for less than ten companies in some industries, the total number of companies we analysed is 50, 80, 115 and 131, respectively. The risk free rate of return is 1-month T-bill returns from French's web site. In the asset allocation stage, Table A6.1 shows that MV is superior for all four industry portfolios and every performance measure. Overall the performance measures in Table A6.2 indicate that, as hypothesised, MV-1/N is superior for all four Fama-French portfolios. 1/N-MV and MV-MV are the worst performers.

Performance Measure		FF = 5	FF = 10	FF = 12	FF = 17
CER	MV	0.0718	0.0741	0.0673	0.0561
	1/N	0.0640	0.0618	0.0604	0.0411
Sharpe Ratio	MV	0.6960	0.7178	0.6631	0.5871
	1/N	0.6404	0.6247	0.6147	0.5158
Dowd Ratio	MV	0.4759	0.5037	0.4464	0.3736
	1/N	0.4202	0.4059	0.3968	0.3080
Omega Ratio	MV	1.8853	1.9506	1.8588	1.7531
	1/N	1.8052	1.8014	1.7903	1.6422

Table A6.1: Performance of 1/N and MV in Forming Portfolios of Four Fama and French Industry Portfolios, i.e. Stage One Asset Allocation - 12-month expanding estimation window (24 months for FF = 17), January 1994 to August 2017, λ = 5, Transaction costs = 50 bps.

	MV Model	λ	η	Initial Window	Constraints	Performance Measure	1	2	3	4
							MV-1/N	1/N-1/N	MV-MV	1/N-MV
1	Bayes-Stein FF=5	5	0.15	12	VBCs	CER	0.0851	0.0833	0.0639	0.0665
						Sharpe Ratio	0.7919	0.7768	0.6363	0.6565
2	Bayes-Stein FF=10	5	0.15	12	VBCs	CER	0.0924	0.0889	0.0841	0.0826
						Sharpe Ratio	0.8696	0.8189	0.8067	0.7830
3	Bayes-Stein FF=12	5	0.15	12	VBCs	CER	0.0886	0.0841	0.0749	0.0775
						Sharpe Ratio	0.8355	0.7815	0.7313	0.7410
4	Bayes-Stein FF=17	5	0.15	24	VBCs	CER	0.0756	0.0722	0.0645	0.0686
						Sharpe Ratio	0.7449	0.7124	0.6566	0.6834

Table A6.2: Performance Measures for the Four Two-stage Procedures for the Four Fama French (FF) Industry Portfolios - 12-month Expanding Estimation Window (24 months for FF = 17), January 1994 to August 2017, λ = 5, Transaction costs = 50 bps.

A7 International Data

As a further robustness check, we repeated the two stage methodology of our core analysis on international data for the UK, USA, Germany, Switzerland, France, Canada and Brazil. This consists of value-weighted total return equity market indices for seven countries - UK (FTSE 100), US (S&P500), Germany (DAX 30), Switzerland (SMI), France (CAC 40), Canada (S&P/TSX Composite), and Brazil (Bovespa); with monthly data from December 1994 to August 2017 expressed in $US. We also analysed the ten companies with the largest market capitalization in each index in December 1994; so in total we have 70 companies. We used 1-month T-bill returns from Ken French?s web site as the riskless rate.

Table A7.1 shows that for asset allocation across these countries MV remains dominant, while Table A7.2 indicates that 1/N is generally preferred for stock selection, although not for Switzerland or Brazil. Table A7.3 confirms our main hypothesis, as MV-1/N is the best strategy for selecting international portfolios. We also find that, on balance, its reverse, 1/N-MV, is the worst strategy.

Performance Measure	1/N	MV
CER	0.0126	0.0272
Sharpe Ratio	0.4178	0.4233
Dowd Ratio	0.2328	0.2398
Omega Ratio	1.4994	1.5305

Table A7.1: Performance of 1/N and MV in Forming Portfolios of the Seven Countries, i.e. Stage One Asset Allocation - 12-month expanding estimation window, December 1994 to August 2017, $\lambda = 5$, $\eta = 0.15$, Transaction costs = 50 bps.

Countries	CER		Sharpe Ratio		Dowd Ratio		Omega Ratio	
	1/N	MV	1/N	MV	1/N	MV	1/N	MV
UK	0.0446	0.0345	0.5480	0.4632	0.3326	0.2690	1.6889	1.5755
US	0.0731	0.0615	0.7182	0.6275	0.4989	0.4117	1.9767	1.8392
Germany	-0.0444	-0.0495	0.3660	0.2818	0.1951	0.1442	1.4163	1.3261
France	0.0062	0.0144	0.4599	0.4021	0.2603	0.2229	1.5338	1.4755
Canada	0.0619	0.0535	0.6736	0.6181	0.4388	0.3897	1.8320	1.7582
Switzerland	0.0113	0.0432	0.4609	0.5274	0.2617	0.3173	1.5487	1.6300
Brazil	-0.2499	-0.1657	0.5368	0.5759	0.3077	0.3386	1.5987	1.6363

Table A7.2: Performance of 1/N and MV in Forming Portfolios of the Shares Within Each County, i.e. Stage Two Stock Selection - 12-month expanding estimation window, December 1994 to August 2017, $\lambda = 5$, $\eta = 0.15$, Transaction costs = 50 bps.

Performance Measure	1	2	3	4
	MV-1/N	1/N-1/N	MV-MV	1/N-MV
CER	0.0632	0.0510	0.0590	0.0552
Sharpe Ratio	0.6512	0.6232	0.6110	0.6117
Dowd Ratio	0.4244	0.3922	0.3928	0.3863
Omega Ratio	1.8318	1.7642	1.7783	1.7554

Table A7.3: Performance Measures for the Four Two-stage Procedures for the International Data - 12-month Expanding Estimation Window, December 1994 to August 2017, $\lambda = 5$, $\eta = 0.15$, Transaction costs = 50 bps.

Country	Constituents
UK - FTSE100	Barclays Bank, British Petroleum, Unilever, GlaxoSmithKlein, British American Tobacco, Diageo, Rio Tinto, Royal Dutch Shell, British Telecommunications, Marks & Spencer
USA - S&P500	Walmart, Exxon-Mobil, Coca-Cola, IBM, General Electric, Proctor & Gamble, Merck, Pepsico, Altria, Bristol Myers Squibb.
Germany - DAX 30	Deutsche Bank, BMW, Allianz, Siemens, BASF, Bayer, RWE, Munich Re, E.ON, ThyssenKrupp.
France - CAC 40	BNP Paribas, L'Oreal, Total, Societe Generale, AXA, Danone, LVMH, Air Liquide, Carrefour, Vivendi.
Canada - S&P/TSX Composite	Royal Bank of Canada, Bank of Montreal, Toronto Dominion Bank, Bank of Nova Scotia, Canadian Imperial Bank of Commerce, Bell Canada Enterprises, Imperial Oil, Barrick Gold, Encana, Thomson-Reuters.
Switzerland - SMI	Nestle, UBS, Roche, Credit Suisse, Novartis, ABB, Zurich Insurance, Richemont, Swiss Re, Swatch.
Brazil - Bovespa	Vale, Petrobras, Companhia Siderúrgica Nacional, Usiminas, Eletrobras, CEMIG, ITAU Unibanco, Banco do Brazil, Bradesco, Lojas Americanas

Table A7.4: Largest Ten Companies in the Seven Country Indices

A8 UK Data

We analysed monthly value-weighted total returns from DataStream on ten UK industry indices, and the ten largest firms in each sector in January 1994. Some sectors had less than ten firms, and so the total number of firms analysed is 56. Our data is from January 1994 to August 2017, and the risk free asset is the Thomson Reuters UK Government Benchmark Yield 1 Month. Table A8.1 shows that in the first stage MV produces better out-of-sample asset allocation results on all four performance measures, as expected.

Performance Measure	1/N	MV
CER	0.0692	0.0783
Sharpe Ratio	0.6017	0.6658
Dowd Ratio	0.4050	0.4725
Omega Ratio	1.8550	1.9519

Table A8.1: Performance of 1/N and MV in Forming Portfolios of the 10 UK Industrial Sectors, i.e. Stage One Asset Allocation - 24-month expanding estimation window, January 1994 to August 2017, $\lambda = 5$, $\eta = 0.15$, Transaction costs = 50 bps

In Table A8.2 1/N produces better out-of-sample results on all four performance measures for the second stage (stock selection) for seven industries, and superior performance by 1/N on three measures for two or three industries. This supports the hypothesis that 1/N is preferable for stock selection.

Table A8.3 compares the overall performance of the four strategies. MV-1/N is the best on all four performance measures and 1/N-MV is the worst, as expected.

Industries	N	CER		Sharpe Ratio		Dowd Ratio		Omega Ratio	
		1/N	MV	1/N	MV	1/N	MV	1/N	MV
Basic resources	6	0.0525	0.0525	0.6789	0.6722	0.4497	0.4438	1.8983	1.8994
Consumer discretionary	10	0.0510	0.0426	0.6259	0.5226	0.4040	0.3207	1.8495	1.6960
Consumer products	8	0.0271	0.0267	0.6222	0.4990	0.3946	0.2986	1.8018	1.6777
Energy	6	-0.0154	0.0072	0.4326	0.4200	0.2455	0.2396	1.5395	1.5416
Financial services	10	0.0572	0.0402	0.5318	0.4106	0.3379	0.2432	1.7478	1.6036
Industrial goods	10	0.0177	0.0012	0.4236	0.2911	0.2438	0.1568	1.5796	1.4381
Real estate	10	0.0248	0.0262	0.4083	0.3824	0.2351	0.2187	1.5661	1.5466
Technology	4	-0.0118	-0.0152	0.5304	0.5140	0.3163	0.3036	1.6772	1.6578
Telecommunications	3	-0.0241	-0.0137	0.3767	0.3471	0.2076	0.1896	1.4910	1.4658
Utilities	5	0.0665	0.0550	0.5977	0.5219	0.3948	0.3288	1.8473	1.7381

Table A8.2: Performance of 1/N and MV in Forming Portfolios of the Shares Within Each UK Industry, i.e. Stage Two Stock Selection - 24-month expanding estimation window, January 1994 to August 2017, $\lambda = 5$, $\eta = 0.15$, Transaction costs = 50 bps.

Performance Measure	1	2	3	4
	MV-1/N	1/N-1/N	MV-MV	1/N-MV
CER	0.0921	0.0853	0.0867	0.0784
Sharpe Ratio	0.7620	0.7185	0.7259	0.6664
Dowd Ratio	0.5652	0.5155	0.5330	0.4705
Omega Ratio	2.1119	2.0455	2.0842	1.9837

Table A8.3: Performance Measures for the Four Two-stage Procedures for the UK Data - 24 month Expanding Estimation Window, January 1994 to August 2017, $\lambda = 5$, $\eta = 0.15$, Transaction costs = 50 bps.

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