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On the long-run solution to aggregate housing systems

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Abstract

This paper explores the properties of dynamic aggregate housing models. In conventional models, in response to demand shocks the primary adjustment mechanism is through prices and changes in housing supply. However, the size of the supply response depends on the price elasticity of supply and in countries such as the UK where the elasticity is low, house prices can rise sharply, worsening affordability. But this ignores the roles of housing risk and credit markets which affect the user cost of capital and the paper demonstrates that models that explicitly introduce a housing risk premium have an additional price stabiliser. The importance is shown through stochastic simulations; these simulations also demonstrate that conventional models used for forecasting and policy analysis may overstate future house price growth.

Keywords

economic processes, finance, housing, affordability, housing risk

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摘要

本文探讨动态集合住宅模型的特征。在传统模型中，为了应对需求冲击，主要的调整机制是通过价格和住房供应的变化。然而，供应反应的规模取决于供应的价格弹性，在英国等弹性较低的国家，房价可能会大幅上涨从而降低可负担性。但这忽略了住房风险和信贷市场的作用，它们影响资本的使用成本，本文证明，明确引入住房风险溢价的模型具有额外的价格稳定因素。重要性通过随机模拟显示；这些模拟还表明，用于预测和政策分析的传统模型可能高估了未来的房价增长。

关键词

经济过程、金融、住房、可负担性、住房风险

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Introduction

This paper is concerned with the long-run growth rate of UK house prices and changes in affordability: between 1969 and 2017 real house prices increased on average by 3.4% per annum – the fastest in Europe – and real household disposable income by 2.7%. Therefore, house prices rose relative to both general consumer prices and incomes. The increases matter not only because of their effect on the wealth distribution – owners have gained relative to renters and younger cohorts – but also because of the implications for macro stabilisation; central banks have paid increasing attention to house price increases because household indebtedness is strongly correlated with changes in prices. Since UK house prices have risen in relative terms, the question arises how house price models provide long-run bounded solutions; this is both a theoretical problem for the literature and a practical problem for house price forecasters. Perhaps surprisingly, this is rarely seen as an issue in the academic literature, whereas predictions produced by the policy community sometimes suggest that affordability will continue to deteriorate over the long run (e.g. Meen, 2011; National Housing and Planning Advice Unit, 2007).

The paper considers how the different stances can be reconciled: two issues turn out to be particularly important; first, some empirical models in the literature suffer from misspecification problems, notably the omission of key variables, which artificially engender a long-run solution by biasing the remaining coefficients of the equations. House price equations based on a discounting relationship sometimes fall foul of the problem. Second, the paper pays greater attention to the role of housing risk and how this may act as a market stabiliser; measures of housing risk are generally included in theoretical models, although they rarely take centre stage. By contrast, risk is not usually incorporated into house price equations used by forecasters; an exception is that the variance of house price changes is sometimes included. In this paper we derive a more formal measure of the housing risk premium consistent with the Consumption (-based) Capital Asset Pricing Model (C-CAPM) obtained from an expected utility version of a lifecycle model often employed in housing economics. A problem for forecasters wishing to use such an approach, however, is that the risk premium can only be projected in a stochastic framework; therefore, standard economic forecasts which do not use stochastic approaches tend to understate the likely levels of housing risk and overstate the expected growth in future

house prices. This is, potentially, an important issue for central banks noting, as above, the greater attention that is now paid to house prices for macro stabilisation.

However, in order to examine these issues, it is helpful to start from an earlier generation of housing models dating back to the 1980s, beginning with the seminal Poterba (1984) model. There is still much to learn from this and other research from the time; this takes us through asset pricing models, lifecycle models and inverted demand function approaches, which are the workhorses of housing economics. In principle all are consistent but differences occur from some of the assumptions and omitted variables. This literature review – also bringing in the later generation of models – is the subject for the second section. The third section introduces risk formally into the lifecycle model and demonstrates how the properties fundamentally change; this is illustrated further in stochastic model simulations in the penultimate section. Conclusions and policy implications are drawn in the final section.

The boundedness of long-run solutions: Why is there a problem?

The nature of the problem

At first sight it is unclear why there is a problem. The most basic models of housing demand and supply suggest that an increase in housing demand, for example arising from an increase in household income, initially leads to a rise in house prices, which overshoot their long-run level because increases in housing construction are modest in the short run, as a result of the required time-to-build. But as supply expands, the long-run increase in prices will be less than in the short run. The expansion in prices relative to the housing stock depends on the price elasticity of housing supply. In the extreme case where the price elasticity of supply approaches infinity, prices return to their initial level (and rise in the long run in line with construction costs). In practice, the supply curve for housing is upward-sloping, although the size of the price elasticity of supply varies internationally. Particularly in countries where there are significant constraints on supply expansion, for example through the land use planning system – and the UK falls into this category – then, even in the long run, the price effect from a demand expansion is strongly positive. Indeed, the strength of the relationship between prices and income is sometimes used as a measure of the weakness of the supply response (Hilber and Vermeulen, 2016). The distribution between price and output expansion is affected by the price elasticity of supply, but the long-run solution is still well-defined.

However, this is a comparative statics analysis that examines the effect of a one-off shock to housing demand; it does not consider the long-run growth path for prices and output where income and other variables are continuously changing over time. To obtain a well-defined long-run growth path, particularly in the reasonable case where supply is not perfectly price elastic, further assumptions are necessary. The widely used Poterba model (1984) is the starting point; this is an asset pricing model originally concerned with the effect of inflation on the price of housing under a non-neutral tax system. In continuous time, the arbitrage equation (1) relates to a condition for the stock of housing, whereas a second equation for new housing construction (2) refers to net additions to the housing stock. (1) is an alternative (and simplified) way of writing the discounting equation for house prices shown in equation (3) below. By re-arrangement of the terms, $g = R(H)/(CC - \dot{g}/g)$ is obtained where the denominator is the appropriate discount

rate. In the numerator, rents are (negatively) related to the size of the housing stock. Furthermore, new construction, (2), is a function of the level of house prices (the validity of this is discussed further below) where b_1 is the price elasticity of supply. The two differential equations can be solved under the assumption of perfect foresight to determine both the level of house prices and the housing stock in the short and long run. Under plausible parameter values, the model satisfies the necessary stability conditions. Three points are important: first, since the model is solved under perfect foresight, the responses to changes occur more quickly than under backward-looking expectations. However, even in this case, Poterba shows that it can take several decades before full adjustment to prices and the dwelling stock takes place under given values for the price and income elasticities of housing demand and the price elasticity of supply. Poterba uses in simulations respective values for the demand elasticities of – 1.0 and 0.75 and estimates values for the price elasticity of supply in the range 0.5 to 2.3. Second, note that Poterba assumes that the income elasticity of housing demand is weaker than the price elasticity and this affects the long-run solution. Below, it is argued that the opposite holds for the UK. Third, although recognising its importance, Poterba (1984: footnote 7, p. 732) states: 'Throughout this discussion, risk and uncertainty play no role in determining the asset price equilibrium.'

$$\dot{g} = - R(H) + CC\, g \qquad (1)$$

$$\dot{H} = b_1\, g - \delta\, H \qquad (2)$$

where: g is the real house price; R is the real rent for housing services; $CC = [i - \pi + \delta + \tau]$ is the real interest rate allowing for depreciation and housing risk; H is the housing stock; δ is the depreciation rate for the housing stock; and $\dot{x} \equiv \frac{dx(t)}{dt}$ denotes the time derivative for any variable $x(t)$.

To emphasise the point, the model still provides a stable long-run solution, although it takes a long time to achieve and the final outcome depends on the demand as well as the supply elasticities.

Discounting price models

As noted, equation (1) is also consistent with a discounting formula where house prices are determined by the discounted present value of the future stream of rental payments, (3), now written in discrete time. The formula provides the starting point internationally for many (but by no means all) more modern models of house prices (see, for example, Black et al., 2006; Miles and Monro, 2019). Empirically, the model is used less frequently in this form in the UK because new data on market rents only became available in 2005, reflecting the fact that private market renting was significant only from the second half of the 1990s. Furthermore, the relationship – originally applied to the pricing of financial assets – needs to be used with some care in the case of housing, since its characteristics differ from financial assets, including the role of transactions costs and housing supply.

$$R(H, RY, HH)_t \,/\, g_t = \left[i_t - \pi_t + \delta_t + \tau_t + \lambda_t - \Delta \ln g_t^e \right] \qquad (3)$$

where: RY is real household income; HH is the number of households; π is the general inflation rate; i is the market interest rate; τ is the housing risk premium; λ is the mortgage constraint measure; and $\Delta \ln g^e$ is the expected real capital gain on housing

There are a number of points to be observed in (3). First, the discount rate in the square brackets is the cost of capital and takes account of the expected capital gain on housing, $\Delta \ln g_t^e$. The relationship also includes a risk premium, τ_t, discussed further

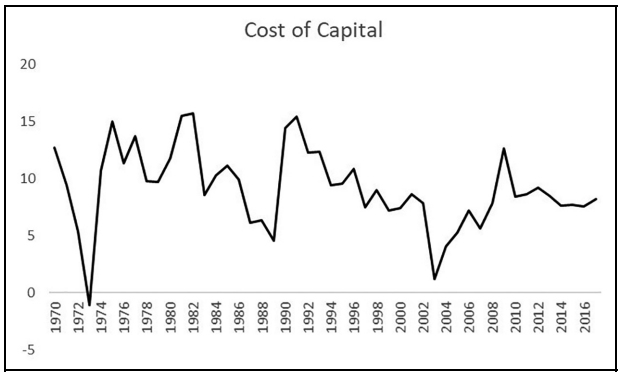

Figure 1. UK cost of capital (%, 1970Q1–2017Q4).
Source: Authors' estimates. See Meen et al. (2016).

below. The cost of capital can also be extended to include the possibility of credit market restrictions, which have become more important in a post-Global Financial Crisis (GFC) world, as many governments have introduced controls on loan to value and loan to income ratios in order to limit credit expansions (see Whitehead and Williams, 2017, for a comparison of the international regulations). The restrictions are denoted by λ_t in equation (3). Credit restrictions represent a shadow price that raises the cost of capital. An operational version of the cost of capital is shown in Figure 1, including a measure of credit constraints, but not a risk premium, extended from Meen et al. (2016: Chapter 10). Second, a version of (3) is used by Himmelberg et al. (2005) to explain the fall in rents relative to house prices in the USA from the 1990s; the relative rise in house prices did not, under this view, reflect a speculative price bubble but a fall in the cost of capital. Gallin (2008) also shows that the US rent to price ratio (the rental yield) can be used to explain future movements in house prices, acting as an error correction variable. On some definitions, the UK also appears to have experienced a fall in the cost of capital from the mid-1980s and, indeed, Miles and Monro (2019) suggest that most of the growth in UK house prices can be explained by a long-run fall in the real risk-free interest rate, $(i_t - \pi_t)$. Nevertheless, over the long run, we might expect the cost of capital to be stationary so that house prices rise at a similar rate to rents, even if it is not yet possible to observe this in the UK data. Third, (3) implies that rents are a (negative) function of the housing stock as in (1) and also (positively) related to real household income, (RY_t), and the number of households (HH_t). Income is generally found to be one of the most important variables explaining house prices and hence is added in (3) but, as noted below, the size of the income elasticity is particularly important.

Bearing in mind the paucity of long-run data on rents in the UK, a testable logarithmic approximation to (3) is given by (4).

$$\ln(g)_t = \gamma_0 + \gamma_1 \ln(RY)_t - \gamma_2 \ln\left[i_t - \pi_t + \delta_t + \tau_t + \lambda_t - \Delta \ln g_t^e\right] + \gamma_3 \ln(HH)_t - \gamma_4 \ln(H)_t \quad (4)$$

(4) forms the basis for many models of house prices in the UK (e.g. Meen, 2013). Although it is not a direct test of the discounting model – it is a joint test with the variables that determine rents – at least in principle it is consistent. (4) can also be used in tests of

Table 1. Estimates of the key elasticities in UK house price equations.

Study	(1)	(2)	(3)	(4)
	Meen and Andrew (1998)	Meen (2013)	Re-estimated on recent data	Column (3), excl. the housing stock
Estimation period	1969 (Q3)–1996 (Q1)	1969 (Q3)–2007 (Q4)	1969 (Q4) –2017 (Q3)	1969 (Q4) –2017 (Q3)
Per household real disposable income	2.401[a]	2.614	2.457[a]	1.210[a]
Housing stock/number of households	−1.744[a]	−1.545	−1.776[a]	–
Cost of capital (semi-elasticity)	−0.04	−0.06	−0.05	−0.09

Note: [a]Specification is slightly different because neither variable is divided by the number of households.

cointegration but, importantly, simply examining the long-run relationship between house prices and income is a misspecification; a finding that the long-run elasticity of house prices with respect to income is unity ties down a long-run solution for affordability – it implies that the price to income ratio has no long-run trend. But the omission of the remaining variables may bias downwards the income elasticity (Meen, 2002), and lead to incorrect policy conclusions. For example, a finding that the income elasticity is one, when the true value exceeds one, is likely to put greater weight on speculative bubbles as the cause of price increases rather than fundamentals. In fact, this turns out to be the case; as shown below in Table 1, estimating (4) – also allowing for lags – between 1969 and 2017, yields an income elasticity of 2.5, but excluding the housing stock reduces the elasticity to 1.2. This is an important point. Bounded long-run solutions for affordability can be obtained in the literature, but this may be the result of model misspecification. If an allowance for housing supply changes is incorporated – as the theory and policy practice suggest – then the income elasticity of house prices is considerably higher.

Inverted demand approaches and the affordability condition

Although (4) may be consistent with the discounting model, UK house price models are not only derived in this way. More simply, the same equation can be derived as an inverted demand function, conditional on the housing stock; Muellbauer and Murphy (1997) is the best known example and remains relevant today. They find that the income elasticity of house prices, γ_1, is approximately 2.5, which is very similar to that in later studies shown in Table 1. The coefficients in (4) can be related to the underlying income elasticity of aggregate housing demand, α_1, and the absolute value

of the price elasticity, α_2. Additionally, α_3 is the elasticity of housing demand with respect to the number of households:

$$\gamma_1 = (\alpha_1/\alpha_2);\ \gamma_2 = 1;\ \gamma_3 = (\alpha_3/\alpha_2);$$
$$\gamma_4 = (1/\alpha_2)$$

Therefore, in the Muellbauer and Murphy model, the income elasticity of housing demand (1.32) is more than twice the price elasticity (−0.52). Note that it is not the absolute value of the two elasticities that matters but the ratio. Some US (and UK) studies have found a ratio closer to one or even less than one, but a key reason for the difference arises from the omission of the housing stock in estimation, which biases the income elasticity of house prices.

The importance of the two parameters can be demonstrated from a long-run affordability equation (6), derived from (5). (5), in turn, is obtained from (4) imposing the coefficient restrictions. Assume also, for the moment, that $\alpha_3 = 1$ (the empirical validity is discussed below), then (4) simplifies to (5).

$$\ln(g)_t = \left(\frac{\alpha_0}{\alpha_2}\right) + \left(\frac{\alpha_1}{\alpha_2}\right)\ln\left(\frac{RY}{HH}\right)_t - \ln\left[i_t - \pi_t + \delta_t + \tau_t + \lambda_t - \Delta\ln g_t^e\right] - \left(\frac{1}{\alpha_2}\right)\ln(H/HH)_t \quad (5)$$

Empirically in the UK, it appears that, approximately in absolute terms, $\left(\frac{1}{\alpha_2}\right) = \left(\frac{\alpha_1}{\alpha_2}\right) - 1.0$. This is demonstrated in Table 1, which shows the values of $\left(\frac{\alpha_1}{\alpha_2}\right)$ and $\left(\frac{1}{\alpha_2}\right)$ from a set of comparable studies, but also shows the effect of omitting the housing stock variable. Under these restrictions, the affordability equation is (6).

$$\ln[g/(RY/HH)]_t = \left(\frac{\alpha_0}{\alpha_2}\right) + \left(\frac{\alpha_1}{\alpha_2} - 1.0\right)\ln\left(\frac{RY}{H}\right)_t - \ln\left[i_t - \pi_t + \delta_t + \tau_t + \lambda_t - \Delta\ln g_t^e\right]. \quad (6)$$

Therefore, for a given cost of capital, housing affordability measured as the ratio of house prices to per household income depends, in the long run, on the ratio of aggregate real income to the housing stock, where the responsiveness depends on the relative sizes of the income and price elasticities of housing demand; for example, using column (3) in Table 1, the coefficient is approximately 1.5. The larger the income elasticity of demand relative to the price elasticity, the greater is the response of prices to a disequilibrium between income and the housing stock. Notice also that, in Table 1, the responsiveness of house prices to a change in the cost of capital is shown as a semi-elasticity, with a coefficient averaging −0.05 across the first three columns, rather than an elasticity as in equation (6) where the expected coefficient is −1.0; this is because under some specifications the cost of capital may take negative values temporarily (see Figure 1).

Equation (6) is, of course, derived under particular coefficient values; there is no necessary reason why they should hold in other countries. Indeed, one of the advantages of the equation is that it highlights possible reasons for differences in potential house price trends. Differences occur not only because of differences in supply (through the price elasticity of supply) but also because of differences in the price and income elasticities of housing demand. Note, however, that the number of households does not appear explicitly in (6). At first sight this might seem surprising and arises partly because of the assumption that

$\alpha_3 = 1$. Table 1 shows results with and without this restriction for different periods and the outcomes are little affected by the restriction. However, this does not imply that the number of households has no effect on affordability; this is because the income measure on the right-hand side is *aggregate* household disposable income, rather than *per household* income. The former is a combination of average household income and the number of households.

Housing supply and other adjustment processes

The divergence between the growth in the housing stock and incomes is related to the price elasticity of housing supply in equation (2), which determines the speed at which the housing stock increases. The greater the elasticity, the faster is the growth rate in the housing stock and the weaker the increase in prices. There are now a large number of studies internationally that attempt to measure the price elasticity of supply. Caldera and Johansson (2013) provide a comparison across 21 OECD countries, finding the price elasticity to be higher in North American and some Nordic countries but lower in other European countries including the UK. This raises an important point of methodology. In (2), it is the *level* of house prices that determines new construction, but Ball et al. (2010) show that the *rate of change* of house prices has a stronger empirical effect in the UK, USA and Australia. This is because in these three countries house prices have a strong long-run upward trend, whereas indicators of construction, such as housing starts, have little trend. Therefore, empirically, the orders of integration of new construction and the growth in prices are consistent, but the elasticity of construction with respect to price levels is likely to be weaker. This limits the ability of new supply to provide price stabilisation. In the simulations in the penultimate section, a price elasticity of supply of one is chosen as the baseline and compared with an alternative case where the elasticity is two. As noted above, these are within the range used by Poterba for the USA, but the lower end is closer to UK empirical results (Ball et al., 2010). As the simulations will show, the effects on affordability of the different assumptions are significant and changes in new construction contribute to price stabilisation but they are not the full story. As (6) indicates, other factors are also at work; from the definition of the cost of capital, these include changes to the risk premium and the possible introduction of mortgage credit controls. In principle, the cost of capital is also affected by property taxation (but, in the UK, governments have been reluctant to use the instrument as a housing market regulator) and more general monetary policies, which have strong housing effects but are geared towards wider macro objectives. The fact that real house prices have risen over time and relative to income might suggest that none of the instruments have been fully effective.

The lifecycle model and the risk premium

In summary, in standard housing models a well-defined solution for the affordability condition requires: (i) a high price elasticity of housing supply so that housing supply grows in line with income; (ii) a relatively low income elasticity of demand compared with the price elasticity of demand; (iii) changes in the cost of capital. None appears entirely consistent with the evidence in the UK. Models in the literature have well-defined solutions but, in some cases, these solutions are obtained either from implicit unrealistic assumptions concerning the price elasticity of supply, or are influenced by omitted variables that bias the estimated

income elasticity of housing demand relative to the price elasticity. The question, therefore, arises whether there are alternative empirically valid specifications in line with theory which ensure a well-defined outcome. Potentially housing market risk and credit constraints can play a role through the cost of capital.

Even early models recognised the importance of housing risk in an asset pricing framework. However, such risk was often not the central focus of attention. Two issues arise: first, whether the risk premium should be time-varying or, as a reasonable approximation, can be treated as a constant; second, whether a theoretically coherent measure of the risk premium can be derived, parallel to that used in financial economics for financial assets. The model in this section indicates that an assumption of risk constancy misses an important element in the housing adjustment process, but the derivation of a theoretically consistent, time-varying housing risk premium is not straightforward. However, once derived, the interpretation of the measure is intuitive. In addition to the Himmelberg et al. (2005) and Black et al. (2006) models mentioned earlier, house price models that explicitly model risk include Campbell et al. (2009), Fairchild et al. (2015), Favilukis et al. (2017) and Jordà et al. (2019). In some cases, for example Black et al., the risk premium is implemented empirically from the variance of the housing return, based on VAR projections. Favilukis et al. is rather closer to the approach in this section; they derive similar optimal conditions to those below, linking the (expected) marginal utility of consumption and the (expected) marginal utility of housing to the (expected) returns on housing and financial assets, but define a housing risk premium as the excess of the former returns over the latter, which differs from that derived here.

This section obtains a housing risk premium from a lifecycle model which is also consistent with the asset pricing and inverted demand function approaches discussed above. The model employs a three-asset case where households can invest in (risky) housing, a risky financial asset or a safe financial asset. This three-asset expected utility model is, in fact, a more general version of the Consumption-based Capital Asset Pricing Model, (C-CAPM) (see Case et al., 2010; Lucas, 1978; Piazzesi et al., 2007). If, for simplicity, the flow of housing services is proportional to the demand for the housing stock (H^d) and, given an assumed constant real discount rate (r), lifetime expected utility is described in infinite horizon discrete time by equation (7):

$$E[U] = E\left[\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \mu\left[H_t^d, C_t\right]\right] \quad (7)$$

$\mu\left[H_t^d, C_t\right]$ denotes the period t utility of the representative household and C_t represents aggregate (non-housing) consumption. (7) is maximised with respect to the budget constraint (8) and technical constraints (9)–(11), which describe changes to real asset stocks (housing and financial, respectively) over time.

$$C_t + g_t X_t + p_t AP_{1t} + AP_{2t} = (1-\theta_t)RY_t + (1-\theta_t)D_t A_{1t-1} + (1-\theta_t)i_t A_{2t-1} \quad (8)$$

$$X_t - \delta_{t-1} H_{t-1}^d = H_t^d - H_{t-1}^d \quad (9)$$

$$AP_{1t} = A_{1t} - A_{1t-1} \quad (10)$$

$$AP_{2t} - \pi_{t-1} A_{2t-1} = A_{2t} - A_{2t-1} \quad (11)$$

where: AP_1 is purchases of units of the risky financial asset; AP_2 is purchases of risk-free assets net of mortgages advances; p is the real market price of the risky financial asset; D is the real dividend per unit of the risky

financial asset; A_1 is the stock of units of the risky financial asset; A_2 is the stock of risk-free assets net of mortgages; θ is the household marginal tax rate; i is the risk-free interest rate; g is the real purchase price of dwellings; X is purchases of dwellings; RY is real household earnings; δ is the depreciation rate on dwellings; and π is the general inflation rate.

In (8), financial assets (net of mortgage loans) are partitioned into the risky (A_1) and risk-free (A_2). Units of the former have a variable real market price (p). The risk-free asset yields an interest rate (i); the equations of motion (10)–(11) also allow for the partitioning of the financial assets. Notice that the model does not include transactions costs, which are typically higher in housing than in financial markets; in this case, this simplification does not affect the key properties of the system, which is concerned with long-run solutions. Arguably, transactions costs primarily affect the speed of market adjustment to an equilibrium, for example, introducing house price autocorrelation. Note also that, in order to keep the study focused on the housing risk premium, possible credit market constraints are omitted but the analysis can be readily extended.

Equations (12) and (13) give the first order conditions.[1] Equation (13) results from the combination of the first-order conditions for the risky financial asset and the risk-free asset. Equations (14) and (15), derived by combining (12) and (13), show the generalised form of the expected marginal rate of substitution between housing and non-housing consumption. Furthermore, since the marginal rate of substitution is equal to the real rent, $R(H, RY, HH)$, the equation is also consistent with (3). In (15), the real capital returns on housing and the risky financial asset are denoted by $r_{h\,t+1}$ and $r_{a\,t+1}$ respectively, where the former is analogous to $\Delta \ln g_t^e$ in (3) above.

$$\frac{E\left[\mu_{H_t^d}\right]}{E[\mu_{C\,t}]} \approx g_t \left\{(1-\theta_t)i_t - \pi_t + \delta_t - \frac{E[\mu_{C\,t+1}\, r_{h\,t+1}]}{E[\mu_{C\,t+1}]}\right\} \tag{12}$$

$$(1-\theta_t)i_t - \pi_t = \frac{E[\mu_{C\,t+1}\, r_{a\,t+1}]}{E[\mu_{C\,t+1}]} \tag{13}$$

$$E[\mu_{Hdt}/\mu_{Ct}] = g_t \quad [(1-\theta_t)i_t - \pi_t + \delta_t - E[r_{h\,t+1}] + \tau_t] \tag{14}$$

$$\tau_t = \{E[r_{a\,t+1}] - [(1-\theta_t)i_t - \pi_t]\} \frac{Cov(\mu_{C\,t+1},\, r_{h\,t+1})}{Cov(\mu_{C\,t+1},\, r_{a\,t+1})} \tag{15}$$

where:

$$r_{h\,t+1} = [g_{t+1} - g_t]/g_t$$

$$r_{a\,t+1} = [p_{t+1} + (1-\theta_{t+1})D_{t+1} - p_t]/p_t$$

τ_t is the key measure of housing market risk in (15) and so (14) provides an explicit form for the housing user cost of capital including risk. Note that this differs from the standard C-CAPM model, which, since the latter does not include housing, fails to capture the covariance term in the numerator and defines a risk premium for risky financial assets rather than housing. These features of our model are novel and distinguish it from the standard C-CAPM, as well as from most conventional housing models.

To be able to derive analytical results, two additional assumptions are needed. First, since future prices and returns are unobservable, households' expectations about the returns on housing, $r_{h\,t+1}$, and risky financial assets, $r_{a\,t+1}$, are assumed to be jointly normally distributed, where $\sigma^2_{h\,t+1}$, $\sigma^2_{a\,t+1}$ and $\sigma_{ah\,t+1}$ in (16) are the variances and covariance of the respective expected returns.

$$\begin{bmatrix} r_{h\,t+1} \\ r_{a\,t+1} \end{bmatrix} \sim N\left(\begin{bmatrix} r^e_{h\,t+1} \\ r^e_{a\,t+1} \end{bmatrix}, \begin{bmatrix} \sigma^2_{h\,t+1} & \sigma_{ah\,t+1} \\ \sigma_{ah\,t+1} & \sigma^2_{a\,t+1} \end{bmatrix} \right) \tag{16}$$

Second, a specific utility function is needed. Utility is, initially, assumed to take the commonly used Constant Absolute Risk Aversion (CARA) form, given by (17) and then extended to the alternative Constant Relative Risk Aversion (CRRA) case.

$$\mu\left[H^d_t, C_t\right] = -e^{-\varphi H^d_t} - e^{-\varphi C_t} \tag{17}$$

where φ is the Pratt-Arrow parameter of risk aversion. On this assumption, the risk premium is defined by (18).

$$\begin{aligned} \tau_t = & \left[r^e_{a\,t+1} - (1-\theta_t)i_t + \pi_t\right] \\ & \frac{\rho_{ah\,t+1}\,\sigma_{h\,t+1}}{\sigma_{a\,t+1}} + \\ & + \varphi g_t\, H_t\left(1-\rho^2_{ah\,t+1}\right)\sigma^2_{h\,t+1} \end{aligned} \tag{18}$$

$\rho_{ah} = \frac{\sigma_{ah}}{\sigma_a \sigma_h}$ is the (expected conditional) correlation coefficient between the return on housing and the risky financial asset.

This risk premium has two parts. The first is analogous to the risk premium for an arbitrary asset or portfolio relative to the (mean-variance investor) efficient portfolio. If subscript a denotes a market index (for example, a stock market index) and subscript h denotes a particular asset (housing, in our case), then $\rho_{ah} \cdot \sigma_h/\sigma_a$ is the systematic risk of asset h – the Sharpe beta. Note that the first part of the housing risk premium vanishes and the second part simplifies to $\varphi g_t H_t \sigma^2_{h\,t+1}$ if housing and financial assets are uncorrelated. However, even in this case, the model shows that simply adding the variance of the housing market return as a proxy for risk leads to misspecification in empirical work.

Equation (18) implies that:

(i) as absolute holdings of housing wealth rise – whether through changes in house prices or the housing stock – housing market risk increases, raising the cost of capital and pushing down house prices; thus, the model has an additional inbuilt stabiliser;
(ii) the cost of capital is positively related to the Pratt-Arrow parameter of risk aversion, φ;
(iii) the housing risk premium is positively related to the correlation between housing and risky financial assets and to the variance of housing capital returns, but negatively related to the variance of financial returns.

The alternative assumption of Constant Relative Risk Aversion (CRRA) utility is shown in (19), where the degree of risk aversion is positively related to the size of the parameter, γ. This modifies the definition of the risk premium, which is now given by (18′).

$$\mu\left[H^d_t, C_t\right] = \frac{H^{d\,1-\gamma}_t}{1-\gamma} + \frac{C_t{}^{1-\gamma}}{1-\gamma} \tag{19}$$

$$\begin{aligned} \tau_t \approx & \left[r^e_{a\,t+1} - (1-\theta_t)i_t + \pi_t\right] \\ & \frac{\rho_{ah\,t+1}\,\sigma_{h\,t+1}}{\sigma_{a\,t+1}} + \frac{\gamma\, C^{e-\gamma-1}_t}{E[\mu_{C\,t}]} \\ & g_t\, H_t\left(1-\rho^2_{ah\,t+1}\right)\sigma^2_{h\,t+1} \end{aligned} \tag{18′}$$

where C^e_t denotes expected consumers' expenditure. The difference from the CARA case occurs in the last term. If the averaging implied by the expectations operator is abstracted from, then $\frac{\gamma C^{e-\gamma-1}_t}{E[\mu_{C\,t}]} = \frac{\gamma}{C_t}$. Importantly, the higher the proportion of the wealth portfolio held as housing relative to consumption, $g_t H_t/C_t$, the greater is housing market risk. Alternatively, if the long-run trends in house prices and the housing stock exceed that of consumption, the risk premium is likely to rise over time.[2] This theoretical result is analogous to the ratio of the real relative quantities of housing services and consumption derived in Piazzesi

et al. (2007). Indeed, the equation can be derived as a special case of the model in Piazzesi and Schneider (2016). The simulations in the next section concentrate on the CRRA case.

Simulation design and results

From (18) and (18′), the risk premium depends on the correlation between the real returns on housing and the risky financial asset, here taken to be an index of UK stock market prices. The latter exhibits more volatility than the former, but between 1970 and 2014 as a whole, the correlation in returns is weak.[3] However, this arises from periods in which the correlation was positive, cancelled by those in which the correlation was negative. Therefore, the precise definition of the sub-periods clearly matters. Nevertheless, since the overall correlation is low, as noted above, this simplifies the risk premium in (18) and (18′) and allows the simulations to concentrate on key model properties and the differences from standard housing models.

Note that the derivation of the risk premium implies that standard house price forecasts and policy simulations conducted within government and the private sector cannot be adequately carried out using deterministic projections of the exogenous variables, notably income, which Table 1 showed to be a key driver of house prices. This is because in deterministic simulations, if income growth is set to its trend, then $\sigma^2_{h\,t+1} \rightarrow 0$ and the risk premium disappears. House price growth is, therefore, likely to be overestimated.

Baseline calibration of the key parameters used in simulation

To summarise, the model consists of three equations – a house price equation, the definition of the risk-adjusted cost of capital, equation (18′), and an equation for new housing supply, analogous to (2), where the key price elasticity of supply is assumed to take a value of either 1.0 or 2.0 in simulations; the risk premium uses the reasonable simplifying assumption that $\rho_{ah} \approx 0$, which also allows the simulations to highlight the model properties in their simplest form. The definition of the cost of capital also allows for possible credit constraints, although these are not the focus here and brief comments are reserved for the concluding section. For the price equation, estimates are needed of the elasticities of house prices with respect to income, with respect to the housing stock and with respect to the cost of capital; these are based on the findings across the studies in Table 1 and take values of 2.5, -1.5 and -0.05, respectively. This implies that the simulations use a *long-run* house price relationship, rather than allowing for the additional short-run dynamics, usually found to be important in house price studies (e.g. Meen, 2013). This allows the paper to explore the solutions arising from the nature of risk, rather than, for example, from price lags which occur from housing market transactions costs.

The simulations below illustrate the different solutions for the variables arising from incorporating housing risk compared with solutions excluding the housing risk premium. The latter are more typical of projections undertaken by housing practitioners.[4] In addition, the simulations show how the variables evolve following unexpected large or persistent shock sequences in the growth rate of real income, resembling the changes that occurred in the Great Moderation (GM) followed by the Global Financial Crisis (GFC). The quarterly growth rate in real household income is estimated as a first-order autoregressive process given in the legend to Table 2. In simulation, temporary shocks are drawn from a log-Normal distribution for the innovations in the estimated stochastic process. Consumption, used in

Table 2. Calibration of key values used in the simulations.

Parameter	Interpretation	Value	Rationale
$(1-\theta)i$	Net-of-tax nominal mortgage interest rate (pa)	0.1047	UK sample average
π	Inflation rate (pa)	0.0578	UK sample average
δ	Housing depreciation rate (pa)	0.01	Assumed value
$RY(0)$	Initial value of households' real income	203,324	Base period (2000Q1)
$C(0)$	Initial value for household consumption	199,319	Base period (2000Q1)
$H(0)$	Initial value of the housing stock	17,039	Base period (2000Q1)
r_RY	Quarterly growth rate of real income	0.006756	UK sample average
r_C	Quarterly growth rate of real consumption	0.006756	Assumed to grow at the same rate as income
σ_RY	SD of the growth rate of real income	0.016228	UK sample statistic

Note: The estimated AR(1) process for real income growth is: $ry_t = 0.008615 - 0.254524 * ry_{t-1} + e_t$, with the empirical distribution of the error term approximately distributed as $N(0, 0.015629^2)$. The expected variance of the house price return, $\sigma^2_{h\,t+1}$ is calculated as a weighted average of expected value in the previous time period (weight = 0.9) and the actual variance calculated over the previous four time periods (weight = 0.1), so the weighting introduces a degree of smoothing into the simulations.
When reproducing the GM-GFC shock sequence in the simulations, the following assumptions are made: a persistent positive shock to real income growth that lasts for 60 quarters (quarters 81–140) – similar to the Great Moderation (GM) – is imposed equal to 20% of the sample mean; a large negative shock that lasts for 8 quarters (quarters 141–148) – similar to the Global Financial Crisis (GFC) – is, then, assumed to reduce real income by 2 standard deviations. Further details and the complete replication of the results are available via MATLAB simulation codes, provided upon request.

(18′), is taken as an exogenous variable, assumed to rise in line with the long-run growth rate of income. Table 2 reports starting values, means and standard deviations for the main variables.

Model sensitivity and simulation of the GM-GFC period

In interpreting the results, the key condition (6) needs to be remembered; under the above coefficient values, the (log) change in affordability is given by (20):

$$\Delta \ln\left[\frac{g}{(RY/HH)}\right]_t = 1.5\,\Delta\,\ln\left(\frac{RY}{H}\right)_t - 0.05\,\Delta\left[i_t - \pi_t + \delta_t + \tau_t + \lambda_t - \Delta \ln g_t^e\right] \quad (20)$$

Therefore, affordability worsens if the growth in income relative to the housing stock exceeds the increase in the cost of capital, weighted by their respective coefficients. Furthermore, through the housing stock, affordability depends on the price elasticity of supply and, as noted above, the sensitivity to alternative values is simulated. However, since the risk premium, τ_t, is positively related to the market value of the housing stock, any initial increase in house prices induced by the income rise will be at least partly offset by the rise in τ_t.

Figure 2 turns to stochastic simulations. The four frames plot the realisations for the key variables – income, affordability, the risk premium and the cost of capital – simulated over 200 quarters;[5] the results are averaged from a thousand replications and are, initially, calculated for a price elasticity of housing supply of 1.0. The stochastic real income growth rate has been calibrated to its mean and standard deviation over the historical period (the results under non-

stochastic income growth are added for comparison). In addition, the effects of the persistent positive income growth in the GM period, followed by the abrupt slump in the GFC, are approximated. To replicate the GM-GFC dynamics, a sequence of two simulated shocks in income growth are introduced: the first is positive and not large in magnitude but persistent (resembling the GM) and the second is large and negative but transient (resembling the GFC).[6] These two shocks are superimposed onto the otherwise mild stochastic setting for the income growth rate. Figure 2 shows three cases: (i) non-stochastic income grows at its long-run trend; (ii) income grows stochastically, but there is no housing risk premium in the cost of capital; (iii) income grows stochastically and the cost of capital includes the risk premium.

The effects of the GM-GFC period are evident in the top-left panel for income. Since the elasticity of house prices with respect to income exceeds two (Table 1), unsurprisingly, the addition of the income cycle produces a strong affordability cycle and is broadly consistent with that observed since the mid-1990s. The impact of the income shocks can be seen by comparing the stochastic and non-stochastic cases for affordability. However, the trend in affordability is noticeably weaker, under stochastic income, once the risk premium is included in the cost of capital. By the end of the simulation period, the risk premium is approximately 2 percentage points, although this rises sharply, temporarily, following the GFC. The modest[7] upward trend in the risk premium reflects the fact that the share of housing relative to consumption (and other assets) is rising over time, increasing its riskiness in the portfolio.

Figure 3 repeats the simulation under a price elasticity of supply of 2.0 and compares affordability and the risk premium under the two cases. A doubling of the price elasticity is, in fact, a large change and well outside the UK experience[8] (although not that of some other countries). As expected, the increase in the growth of the housing stock – were it to be achieved – would lead to considerably improved affordability and to a lower risk premium. The point, therefore, is that a combination of influences affect affordability in the longer run, but if it were to be improved by supply expansion alone, the required increases would need to be very large relative to history. The formal incorporation of housing risk, derived from the theoretical framework, adds a further dimension to the analysis of long-run housing dynamics.

In conclusion: Implications for policy

The paper provides a number of lessons for modelling, forecasting and policy. The long-run solution to house price models is not an issue that has attracted much attention in the literature recently. At first sight, there does not appear to be a problem; even early generations of housing models have well-defined solutions and meet the necessary stability conditions. Later generations of models, based on asset pricing, also appear to have clear solutions, stressing the long-run relationship between house prices and incomes; affordability cannot worsen forever because mortgage payments would take up an increasing share of income. However, the paper has suggested that there are problems both for the academic literature and for housing practitioners/forecasters. The solutions proposed in the literature often rely on implausible values for the income and price elasticities of housing demand (at least in the UK case) and may arise from model misspecifications. Notably, the omission of supply variables in some asset pricing models,

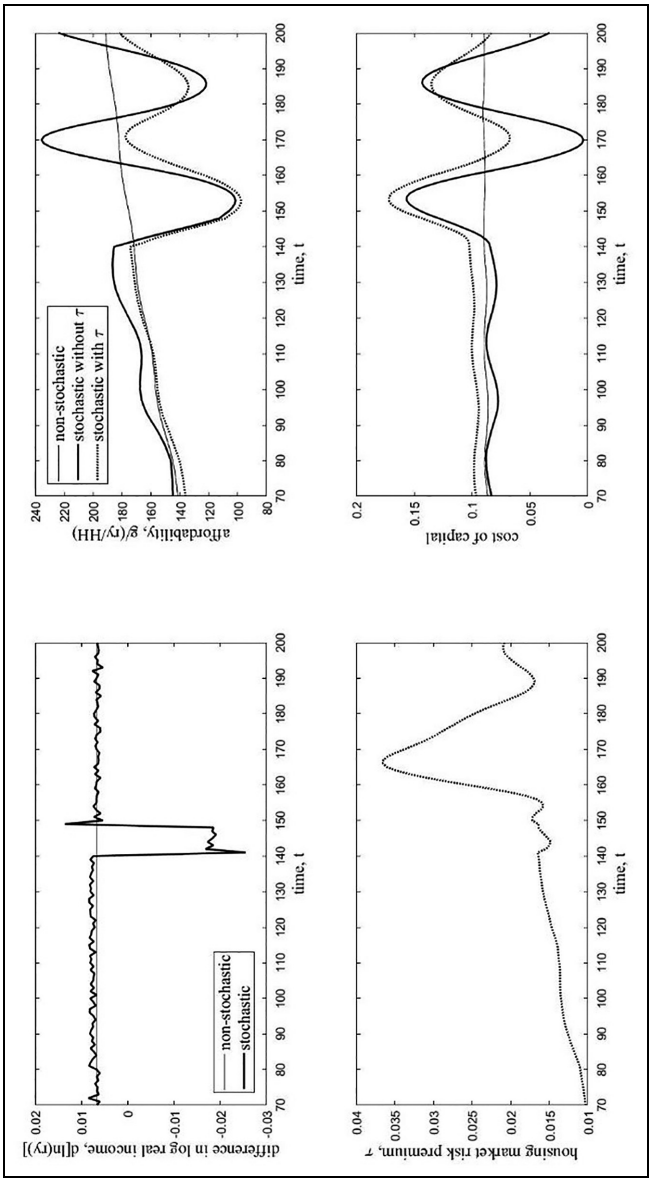

Figure 2. Average dynamics of 1000 stochastic replications imposing two large shocks to simulate the GM-GFC boom-bust cycle in UK house prices.

Figure 3. The sensitivity of affordability and housing risk to alternative price elasticities of supply.

biases downwards the estimated income elasticity of house prices and artificially produces a misleading long-run solution.

Therefore, the question addressed here is whether there are additional factors that contribute to a more stable outcome. The paper concentrates on the role of the housing risk premium, which is generally recognised as relevant in house price models that include the cost of capital, but its role is often underplayed. The paper contributes by formally defining the cost of capital in a C-CAPM housing framework and identifies the key factors. Once risk is taken into account, then housing markets have an additional built-in stabiliser that prevents price-to-income ratios increasing without bound. Importantly, the risk premium is not adequately captured by the variance of house price changes alone, but also depends on the degree of risk aversion, the market value of the housing stock, the variance in the return on financial assets and the covariance in returns between financial assets and housing. In the case where the covariance is zero, which has approximately been the case in the long run, the definition of the risk premium simplifies. However, the paper demonstrates an issue for forecasters and for policy; most economic forecasts are produced in a non-stochastic setting. Since, over the projection period, the model variables will tend towards their long-run trends, the variance of house price growth (and hence the associated risk premium) tends to zero. Therefore, forecasters are likely to overstate the growth in house prices, even in a growing economy. The issue is probably of less importance in short-run projections, but our simulations show that it is important in the long run.

A final question is whether the housing risk premium is the *only* additional market stabiliser. Conventional changes in housing supply are also important but the paper suggests that, alone, they are insufficient to stabilise price growth given the low price elasticities of supply found in UK studies. In addition, many countries, as part of enhanced macro stabilisation policies since the GFC, have introduced significant controls on mortgage lending through limits on loan to value and loan to income ratios and through the introduction of stress tests. Mortgage institutions have also tightened their own lending criteria. Formally, the introduction of controls that limit borrowing below the level required by households – a rise in deposit requirements is one implication – raises the user cost of capital, since credit restrictions are a form of shadow price (Meen, 1990). Therefore, although outside the scope of this paper, credit controls also act as a policy instrument for market stabilisation, but there are distributional consequences since the controls fall most heavily on first-time buyers.

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Notes

1. Full proofs are available from the authors.
2. Note that, in the CARA case, the trend in the risk premium will be stronger since no account is taken of the trend in consumption.
3. For example, the correlation in real returns calculated over a 2-year horizon is 0.13. The

correlation in quarterly returns is also modest.
4. To be fair, our own forecasts in the past have suffered from the same problem.
5. The graphs only show the results between periods 70–200 as the earlier periods are required for the model to settle towards its long-run solution, which is the primary interest.
6. For more detail, see Table 2.
7. Note the large scale of the vertical axis. The trend would be stronger under an assumption of CARA utility.
8. Over the simulation period, it leads to an average growth rate of the housing stock approximately 50% higher than that observed historically.

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