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David Newton, D.P.Newton@bath.ac.uk, School of Management, University of Bath, Claverton Down, Bath, BA2 7AY, UK, Tel: +44(0) 1225 386890

Emmanouil Platanakis, E.Platanakis@bath.ac.uk, School of Management, University of Bath, Claverton Down, Bath, BA2 7AY, UK. Tel: +44(0) 1274 235311, ORCID: 0000-0003-1493-6339

Dimitrios Stafylas, dimitrios.stafylas@york.ac.uk, York Management School, University of York, York, YO10 5GD, UK, Tel +44(0) 1904 326 437

Charles Sutcliffe, c.m.s.sutcliffe@reading.ac.uk, ICMA Centre, Henley Business School, University of Reading, PO Box 242, Reading RG6 6BA, UK, Tel +44(0) 118 378 6117, ORCID: 0000-0003-0187-487X

Xiaoxia Ye, Xiaoxia.Ye@liverpool.ac.uk, Management School, University of Liverpool, Chatham Street, Liverpool L69 7ZH, UK, Tel +44(0) 151 795 7638

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Abstract

For 5,500 North American hedge funds following 11 different strategies, we analyse the stand-alone performance of these strategies using a stochastic discount factor approach. Employing the same data, we then consider the diversification benefits of each hedge fund strategy when combined with a portfolio of US equities and bonds. We compute the out-of-sample Black-Litterman portfolios, with Bayes-Stein, higher moments, simulations, desmoothed data and allowance for regimes as robustness checks. All but two hedge fund strategies out-perform the market as stand-alone investments; and all but one provide significant diversification benefits. The higher is an investor's risk aversion, the more beneficial is diversification into hedge funds.

1 Introduction

The term hedge fund (HF) has evolved to mean an actively traded portfolio of investments that engage in a wide range of investment and trading activities. They have become attractive investments, especially for long term institutional investors such as pension funds and insurance companies seeking to improve their portfolio performance via diversification into alternative assets. This appeal to long term investors has been important for the development of the HF industry; and in 2017 45% of institutional investors allocated part of their portfolio to HFs, with a total investment of $2.06 trillion (Preqin, 2018).

There are many perspectives to evaluating HFs. For instance, Shawky, *et al.* (2012) show that HFs diversifying across sectors and asset classes outperform their peers by an average of 1.1% per year. Patton (2009) and Bali *et al.* (2012) examine HF market neutrality, and find that even market neutral HFs have some market exposure. Other studies (e.g. Bali *et al.,* 2014; Racicot and Theoret, 2016; Namvar *et al*., 2016) and Savona (2013) are concerned with the effects of systematic risk on HFs; and Patton and Ramadorai (2013), Hubner *et al.* (2015) and Agarwal *et al.* (2017) investigate the effects of volatility on HFs. Other perspectives include the identification of structural breaks in HF returns (Bollen and Whaley, 2009; Giannikis and Vrontos, 2011; Jawadi and Khanniche, 2012; Billio *et al.,* 2012; and Akay *et al.,* 2013); and the use of asset-based factors (Fung and Hsieh, 2004). Another stream of studies (e.g. Do *et al*, 2010; Eling, 2009; Joenvaara, *et al*, 2016; Hentati-Kaffel and Peretti, 2015) examines persistence in HF performance. HFs can create significant abnormalities in portfolio return distributions. HF returns generally have the attractive feature of a low correlation with other securities (Amin and Kat, 2003), but also the unattractive features of negative skewness and excess kurtosis (Brooks and Kat, 2002). The non-normal distribution of HF returns is confirmed by many later studies, such as Wegener *et al.* (2010). Some HF strategies control downside risk by taking long/short positions, and can have a superior risk-return performance to other investments.

“The response of HF strategies to the phase of the business cycle is strongly asymmetric. Racicot and Théoret (2016, 2018, 2019) and Gregoriou et al. (2021) use a wide variety of nonlinear econometric methods to establish this asymmetric behaviour. Racicot and Théoret (2016) adopt the novel methodology of Beaudry et al. (2001) to examine the properties of HF strategies across the phases of the business cycle, and find that their behaviour with respect to macroeconomic uncertainty is highly asymmetric. Racicot and Théoret (2018) use the smooth transition vector autoregressive model of Auerbach and Gorodnichenko (2012), and show that the response of HF strategies to macroeconomic and financial shocks depends on the stages of the economic cycle; with their beta increasing during an expansion, and decreasing during a recession. Racicot and Théoret (2019) employ the Markov regime-switching model of Hamilton (1989, 1994) to show that HF strategies’ higher moments tend to jump during economic slowdowns, and are more stable during expansions. Gregoriou et al. (2020) use the nonlinear projection technique of Jorda (2005) to show that the asymmetric behaviour of HF strategies is much more pronounced during recessions. Chua et al. (2009) and Page and Panariello (2018), among others, also support these results about the asymmetric behaviour of HFs, since both these studies show that left-tail correlations between HFs and other asset classes are higher than right-tail correlations, which questions the inclusion of HFs in multi-asset portfolios during crises.”

HF investment can be as an individual investment, or as part of a portfolio. Since portfolio performance is heavily influenced by asset correlations, as well as their returns and variances, it does not follow that HFs that perform well individually are also good as part of a portfolio. HFs with a good individual performance will have high returns and-or low risk, but may be unsuitable as portfolio diversifiers because they have high correlations with other financial assets. Therefore stand alone and portfolio investors may be attracted to different HFs. To investigate whether some types of HF are dominant both individually and in portfolios, we first we use stochastic discount factors (SDFs) to investigate the performance of different types of HF as stand-alone investments.

Then we use portfolio theory to study whether those types of HF that perform well as individual investments also improve the performance of portfolios of other financial assets.

The performance of different HF strategies as stand-alone investments, relative to other HF strategies and to the stock market, has been studied by a number of researchers (e.g. Agarwal and Naik, 2004; Bali et al., 2013; Capocci, et al., 2005; Capocci and Hubner, 2004; Ibbotson et al., 2011; Patton and Ramadorai, 2013; Stafylas et el., 2018). These authors employ a variety of performance measures (CAPM alpha; manipulation-proof performance measure (MPPM); almost stochastic dominance; and the Sharpe, Calmar, Sortino, Treynor, and mean-VaR ratios). Agarwal and Naik (2004) find that most HF strategies have positive alphas, and Stafylas et el. (2018) show that HF strategies generally have positive alphas in rising or expanding markets. Ibbotson et al. (2011) and Patton and Ramadorai (2013) find positive alphas for about half the HF strategies they study; and using a wide variety of performance measures, Bali et al. (2013) also show that most HF strategies outperform the market. Cao et al, (2018) find that hedge funds tend to invest in shares with a positive alpha, which subsequently outperform the market. However, using Sharpe ratios, Capocci and Hubner (2004) demonstrate that almost all HF strategies are inferior to the market.

The inclusion of HF strategies in portfolios of conventional assets can provide diversification benefits. These benefits have been studied, and the results are mixed. In their influential study Amin and Kat (2003) analyse the monthly returns of 77 HFs and 13 HF indices, and show that 12 HF indices and 72 individual HF funds are inefficient as stand-alone investments. They also find that, when combined with the S&P500, seven HF indices and 58 individual funds create efficient portfolios. However, this study uses an in-sample analysis. Denvir and Hutson (2006) examine the in-sample performance and diversification of 332 funds of HFs (FOHFs). They find that, although the FOHFs underperform a composite HF index, they have lower correlations with stock indices than the composite HF index, making the FOHFs better diversifiers. However, Denvir and

Hutson (2006) do not form portfolios. Signer and Favre (2002) use portfolio theory in an in-sample setting to highlight the diversification benefits of including a HF index in a portfolio. An in-sample analysis by Cumming *et al.* (2014) finds that the inclusion of an index of FOHFs improves the performance of a portfolio of seven other asset classes across a wide range of levels of risk aversion. Jackwerth and Slavutskaya (2016) show that the addition of HFs to UK pension asset-liability portfolios produces significantly higher out-of-sample diversification benefits than other alternative investments. This study uses a portfolio of random HFs, rather than HF strategies, to demonstrate this benefit, and there is no portfolio optimization. Hoevenaars *et al.* (2008) form in-sample portfolios for seven asset classes plus pension liabilities, and find that a FOHF index features in all the optimal portfolios. In contrast to these studies that find diversification into HFs to be beneficial, Platanakis *et al*. (2019a) use 19 portfolio models in an out-of-sample environment, and show that adding the FOHF composite index to a stock-bond portfolio is harmful for investors. The literature generally finds evidence that hedge funds in portfolios of conventional assets provide diversification benefits, although diversification has its limitations.[1]

We evaluate the individual performance of 11 HF strategies. Following Li *et al*. (2016), we use their SDF framework. We find that nine of our 11 HF strategies have positive pricing errors and so out-perform the market, although the implied skill levels of HF managers are small. Three of the four directional HF strategies are the worst stand-alone performers. We then analyse the 11 HF strategies by adding them to a stock-bond portfolio, and investigate whether they add economic value in an out-of-sample setting. This analysis employs four different versions of the Black-Litterman (BL) portfolio model; three different values of risk aversion (λ = 2, 5, and 10) to represent aggressive, moderate and conservative investors respectively; and a rolling window when estimating the various input parameters of the portfolio optimization process. The use of a

[1] Goetzmann et al. (2005) find the correlation of returns is higher when market conditions are worst, making diversification less useful for investors during hard times. Optimized portfolios are very sensitive to input errors (Michaud and Michaud, 2008), and dealing with the non-normality of asset returns, transactions costs and illiquid asset classes, (Kinlaw et al. 2013) also present a challenge for optimized portfolios.

portfolio optimization technique that is robust to estimation risk, such as BL, is important because portfolio theory is highly sensitive to estimation errors[2], and is more important for alternative investments such as HFs (Platanakis *et al.* 2019a).

Our study has seven robustness checks - an alternative type and length of the estimation window (a 12 month expanding window, rather than a 60 month rolling window), a different portfolio optimization technique that is robust to estimation errors (Bayes-Stein), the inclusion of higher moments in the objective function, the use of both a 60 month expanding window and a 12 month rolling window, data simulated from a multi-variate normal distribution fitted to our sample, the use of desmoothed data, and allowance for different regimes to control for effects such as business cycles. We find that all but one of the 11 HF strategies offer diversification benefits, and six are significantly beneficial.

We contribute to the literature in various ways. We are the first to use the same data to study the performance of HFs both individually and in portfolios. Our empirical analysis uses a merged and cleaned database of individual North-American HFs using administrative data (HF name/legal structure, management company/legal structure, manager name, and inception name), and quantitative data (correlations of HF returns). At over 5,500 HFs, this is one of the largest samples of North American HFs. We are the first to use the same data to both compare the individual and portfolio performance of different HF strategies. Our common data set provides a level playing field when comparing their stand-alone and diversification benefits. We are the first to use stochastic discount factors (SDFs) to compare the performance of different HF strategies. The use of SDFs addresses two problems with CAPM alphas: inefficient benchmarks leading to ambiguous rankings (Dybvig and Ross, 1985); and, since hedge portfolios change over time, ignoring the possibility of time varying expected returns and risk (Admati and Ross, 1985). To

[2] Estimation risk has been discussed extensively in the portfolio theory literature (e.g. Kolm *et al.* 2014; Levy and Roll, 2010; Levy and Levy, 2014; Platanakis *et al*. 2019a; Platanakis and Urquhart, 2019; Platanakis *et al*, forthcoming).

estimate the pricing errors we minimize the Hansen-Jagannathan distance while using the MIXIV model specification of Li et al., 2016). HF strategies have different characteristics, and the estimation errors in forecasting portfolio inputs may differ as between HF strategies. Therefore the diversification benefits of the various HF strategies need to be examined out-of-sample using optimized portfolios. Previous studies do not provide a systematic examination of whether including different HF strategies in optimized portfolios offers diversification benefits out-of-sample. We fill this gap by examining the out-of-sample performance of 11 HF strategies as portfolio diversifiers using the out-of-sample performance of portfolios optimized using the Black-Litterman model, with the Bayes-Stein model, higher moments and simulations as robustness checks.

Our analysis provides information to investors on both the stand alone and diversification benefits they can expect from HFs that follow different strategies. They can also benefit from negotiating incentive fees for HF managers based on the strategy's performance as a portfolio diversifier or stand-alone investment. Financial authorities can benefit from our results when setting regulations, such as whether a particular product should be available to retail investors.

This paper is organized as follows: in Section 2 we describe our data, and in Section 3 we present our evaluation of the performance of individual HFs. Section 4 contains our portfolio methodology, with our empirical results in Section 5. Our robustness checks appear in Section 6, with our conclusions in Section 7.

2. Hedge Fund Data

We analyse the EurekaHedge and BarclayHedge HF databases (DBs), which both contain live and dead funds. The inclusion of dead HFs addresses the issue of survivorship bias. We consider net-of-fees monthly returns from January 1995 to March 2014, as most HF databases came into existence in the early to mid-1990s. Many HFs are included in both of these DBs, and so it is necessary to eliminate duplicates. Our algorithm for finding duplicate HFs requires them to have the same name, management company, fund manager and inception date; together with returns with a correlation of at least 0.99[3]. Where there is a match, we eliminate the HF with the shortest return track record (or assets under management (AUM), if the returns track records are of equal length). We treat consecutive zero returns as missing values. Our data set covers HFs that invest mainly in the North American region, or have a reported exposure to this region[4]. The following 11 HF strategies are used in our analysis: commodity trading advisors (CTA), event driven (ED), global macro (GM), long only (LO), long short (LS), market neutral (MN), multi strategy (MS), relative value (RV), sector (SE), short bias (SB) and others (OT)[5]. Table 1 shows how we reduce the initial sample of 31,744 funds to 5,504 funds.

We eliminate the first 12 returns of each HF to minimize the instant history bias[6]. Some authors (e.g. Bali *et al.* 2011; Ibbotson *et al.* 2011) exclude the first 12 monthly returns; while others like Ackermann *et al.* (1999) exclude the first 24 or more monthly returns. The annual median life of the HFs in our sample is six years, which is similar to Gregoriou (2002), and excluding 24 or more monthly HF returns would remove approximately one third of our data. We winsorize our data to deal with outliers, and each month truncate the top and bottom 0.5% percentiles for that month. To avoid our results being dominated by large HFs, we analyse the equally weighted mean returns of the HFs following each strategy[7].

[3] We do not consider share restriction information, the compensation structure or the strategy descriptions due to different reporting standards, and the changing nature of some characteristics.

[4] HFs with a North American focus were defined as follows. For BarclayHedge US-focus funds are those with a *Fund_Geographical_Focus* equal to 'North America' and/or 'Global' with exposure to North America - as denoted by the *Fund_Exposure_North_America* field with at least 50% exposure/investing. For EurekaHedge US-focus funds are those with a *Geographical_Mandate* equal to 'Canada' and/or 'North America', or funds with *Geographical_Mandate* equal to 'Global', with 'USA' and/or 'Canada' and/or 'North America' in the *Country_Focus* field with at least 50% exposure/investing.

[5] This classification of HF strategies is similar to Joenvaara *et al.* (2012). A description of the underlying HF strategies is in the Appendix.

[6] The instant history (or backfill) bias occurs because HF managers are not obliged to report their performance, and only successful HF managers with a good track record have an incentive to publish their performance in a DB. Private DB vendors have inclusion criteria, e.g. 12-24 months of returns; and this leads to an upward bias in the performance of HFs for the period before they enter a DB.

[7] Brown (2016) points out that to reduce risk it is sensible to invest in ten or more hedge funds following a particular strategy. Each of our strategies contains many more than ten hedge funds.

As well as HF returns, for the portfolio diversification analysis we use US dollar returns for the S&P500 composite total return index (code S&PCOMP(RI)), the bond yield of the Barclays US aggregate month-to-date return (code LHAGGBD(TOTR)) and the one-month US treasury bill yield. Table 2 has descriptive statistics for the equity index, bond index, one-month US treasury bill yield, and the annual raw, net-of-fees, returns for the 11 HF strategies under consideration. The SB strategy is risk-return dominated by every other HF strategy and by the bond index and risk-free rate. The equity index is risk-return dominated by CTA, ED, GM, LS, MS, OT; and CTA risk-return dominates GM, MS and RV. Using the Jarque-Bera test we reject at the 1% significance level the null hypothesis that the distributions of the 11 strategies' returns are normal.

Table 1 – Selection of the Sample

	Live HF	Dead HF	Totals
EurekaHedge	7,150	5,865	13,015
BarclayHedge	3,885	14,844	18,729
Totals	11,035	20,709	31,744
Removal of dead CTAs, FoF, MFA*	0	7,406	7,406
Removal of duplicates	156	2,262	2,418
Removal of non US-focused HFs, funds with missing data, or no data after 1994	8,972	7,444	16,416
Totals	1,907	3,597	5,504

* These dead funds were removed from the BarclayHedge DB because it had no live CTAs, while FoFs and MFAs were not included in our analysis. The EurekaHedge DB did not contain any FoFs or MFAs.

Table 2 - Summary Statistics

This table presents summary statistics of the annualized (net-of-fees) returns for each strategy. It also includes descriptive statistics for equity, bonds, and the 1-month treasury yield.

Strategy	Symbol	No. of HFs	Mean	Std. Dev.	Skewness	Kurtosis
CTA	CTA	459	11.87%	0.069	0.046	-1.081
Event Driven	ED	528	12.07%	0.130	-0.982	3.422
Global Macro	GM	113	10.68%	0.089	0.881	-0.265
Long Only	LO	313	14.08%	0.169	-0.985	2.282
Long Short	LS	1991	12.91%	0.138	-0.284	1.218

Market Neutral	MN	228	6.52%	0.047	1.097	0.190
Multi Strategy	MS	227	11.84%	0.102	-0.705	1.648
Relative Value	RV	932	10.26%	0.087	-0.624	3.542
Sector	SE	464	13.35%	0.163	-0.153	1.304
Short Bias	SB	38	1.30%	0.207	1.210	1.298
Others	OT	211	12.95%	0.082	0.008	-0.141
Equity Index	-	-	10.42%	0.154	-0.208	2.841
Bond Index	-	-	6.13%	0.036	-0.063	2.844
Risk-free	-	-	2.75%	0.006	0.006	2.614

Correlations between the 11 HF strategies and the other two assets in our portfolios appear in Table 3. Four strategies have a negative correlation with bond returns (LS, ED, LO and OT), and one (SB) has a large negative correlation with equities. These negative correlations suggest that some HF strategies may have substantial diversification benefits.

Table 3 – Correlations Between Hedge Fund Returns and Equities and Bonds

HF Strategy	CTA	ED	GM	LO	LS	MN	MS	RV	SE	SB	OT
S&P500	0.02	0.722	0.461	0.854	0.794	0.230	0.634	0.609	0.757	-0.782	0.684
1M Treasury Bond	0.274	-0.046	0.196	-0.045	-0.051	0.094	0.029	0.094	0.003	0.093	-0.008

3. Individual Hedge Fund Performance

To see how the 11 HF strategies perform as individual investments, we follow the approach of Li *et al*. (2016) to estimating the skill of HF managers. In the investment performance literature it is well known that inefficient benchmarks can cause ambiguous evaluations (Dahlquist & Söderlind, 1999; and Dybvig & Ross, 1985), and suffer from the joint hypothesis testing problem (Li, et al. (2016). The traditional CAPM alpha approach to investment evaluation is subject to this inefficient benchmark problem, leading to potentially ambiguous rankings. It is also an unconditional measure which ignores time variations in expected returns and risk (Dahlquist &

Söderlind, 1999). More importantly, hedge funds are lightly regulated, and engage in active investment with great trading flexibility (Fung, et al., 2008). Due to their dynamic trading strategies and use of derivatives, traditional linear asset pricing models such as CAPM may give a misleading evaluation of hedge fund performance (Li, et al., 2011).

In view of these issues, we use the Stochastic Discount Factor (SDF) framework for hedge fund performance evaluation developed by Li, et al. (2016) to quantify hedge fund performance. They use a simulation analysis, where the skill of HF managers is known, to choose an appropriate specification for the regression equation used to estimate HF pricing errors (which we define below). They vary manager skill level between zero and one, where the closer skill is to one, the better is the out-performance, relative to zero skill[8]. The regressions to estimate HF pricing errors minimise the Hansen-Jagannathan (1997) (HJ) distance and, as mentioned by Li *et al.* (2016), "the HJ-distance has a nice economic interpretation as the maximum pricing error of all linear payoffs constructed from the primitive assets, R_t, with a unit norm. This makes model comparison based on the HJ-distance economically meaningful." They then use the pricing errors estimated using the simulated data with known manager skill levels to transform pricing errors estimated using actual HF data, to give the corresponding HF manager skill levels.

As our regression specification we use MIXIV, one of the SDF models introduced by Li *et al.* (2016) that performs well in their simulations. This model mixes an option-based model, that accommodates option-like assets such as HFs, with the Fama and French (1993) model. It also allows for time-varying model coefficients, and fits the data better than its unconditional counterpart[9]. Applying the MIXIV model to our HF returns gives expected pricing errors for each

[8] In theory the value of skill lies between zero and one. However, in empirical studies the estimated value of skill can be negative. A negative value of skill indicates that the performance is worse than for a zero-skill manager.
[9] We obtain the option data from WRDS whose time period matches that of our HFs returns data.

of the 11 HF strategies. Then using the skill-pricing error relationship in Table 5[10] of Li *et al.* (2016), we recover the corresponding skill measure for our 11 HF strategies.[11]

More formally, let y_t be the proxy for the true SDF, and be defined as a function of various risk factors and the prices of these risks. In the MIXIV model y_t is defined as:-

$$y_t = (b_0 + b_1 z_{t-1}) + (b_2 + b_3 z_{t-1})MKT_t + (b_4 + b_5 z_{t-1})SMB_t + (b_6 + b_7 z_{t-1})HML_t$$
$$+(b_7 + b_8 z_{t-1})MOM_t + (b_9 + b_{10} z_{t-1})STR_t + +(b_{11} + b_{12} z_{t-1})SKEW_t, \quad (1)$$

where z_t is the one-month T-bill rate which captures time-variation in the price of risk; MKT_t is the monthly return on the value-weighted CRSP index in excess of the one-month T-bill rate; SMB_t, HML_t, and MOM_t are the return differentials between small and large firms, high and low book-to-market firms, and winner and loser firms, respectively; STR_t is the return on at-the-money (ATM) S&P500 index straddles with a time to maturity of between 20 and 50 days which captures aggregate volatility; and $SKEW_t$ is the return on out-of-the-money (OTM) S&P 500 index puts that expire in 20 to 50 days and captures jump risk in the market index[12]. All the data has a monthly frequency, and is from February 1996 to March 2014.

As in Li *et al.* (2016), we consider 16 primitive test assets: six stock portfolios sorted by size and book-to-market ratio to capture cross-sectional return differences due to size and value effects; six stock portfolios sorted by size and past returns to capture size and momentum effects; three option-based assets, ATM calls, ATM puts, and OTM puts on the S&P 500 index; and the

[10] This table reports the monthly abnormal returns (alphas, pricing errors) of simulated long/short equity hedge funds under 10 different levels of manager skill (γ). Therefore, it serves as a mapping between skills and pricing errors.
[11] Although Table 5 in Li *et al*. (2016) is based on long/short equity HFs, we believe the results are representative of, and applicable to, other HF strategies. As pointed out by Li *et al*. (2016) long/short HFs represent the largest number of HFs, and in the past two decades have one of the largest assets under management for HF strategies. In our sample of 5,504 HFs, 36% follow the long/short strategy; which is more than twice the percentage for the next largest strategy.
[12] Data on MKT_t, SMB_t, HML_t, and MOM_t are obtained from Kenneth French's web site. We collect STR_t and $SKEW_t$ from Datastream.

one-month T-bill rate. To evaluate the performance of HFs using different strategies, for each strategy we estimate the SDF model using the equally weighted returns of the HFs following that strategy and the 16 primitive test assets. This estimation produces expected pricing errors for the 11 HF strategies, and these are then compared with those in Table 5 of Li *et al.* (2016) to infer the performance of the HFs in terms of their corresponding skill levels. Since we employ the MIXIV model, we use the skill levels of MIXIV in Panel A of in Table 5 of Li *et al.* (2016).

Estimation of the regression equation to explain y_t is performed by minimizing the distance HJ-distance, which is defined as:-

$$\delta = \sqrt{E\left(\boldsymbol{\alpha}'\right)\boldsymbol{E^{-1}}\left(\boldsymbol{RR}'\right)E(\boldsymbol{\alpha})} \tag{2}$$

where the expected pricing error is $\alpha_t = E_{t-1}(y_t R_t) - 1$, and R_t is the return on primitive assets or the HF. If y_t prices the primitive assets and HFs correctly, then the expected pricing error should be zero for all the primitive assets plus the HFs and their linear combinations. Since none of our estimated expected pricing errors are larger than 0.2, we employ linear interpolation[13] using the values of first two pairs of skill-pricing error values from Table 5 of Li *et al.* (2016) to compute the implied skill levels. The skill levels have an explicit relation with the expected pricing errors in Table 5 of Li *et al.* (2016). We employ this relation to calculate the implied skill level corresponding to the expected pricing errors of the different HF strategies. Due to the way the skill levels are calculated, the statistical significance of the skill levels is not applicable. The expected monthly pricing errors and corresponding skill levels are summarised in Table 4.

[13] Gamma = (alpha + 0.02) / (4.4 + 0.2)

HF strategies LS and SE have negative pricing errors and SE also has negative skill, while the remaining nine strategies have positive skill and pricing errors, i.e. market out-performance. The strategy with the largest pricing error and skill is SB, followed by GM; and their scores are some way larger than those for the other nine strategies. It is interesting that the strategy with the largest positive pricing error (SB) is risk-return dominated by the equity and bond indices, and by all the other HF strategies. These results suggest that, as stand-alone investments, nine of the HF strategies out-perform the market, although their level of skill is low. However, most investors hold HFs as part of a portfolio, so the benefits they bring to a portfolio are generally more important than their stand-alone performance, and we investigate this in the next section.

Table 4 Hedge Fund Performance Evaluation – Alphas and Skill Levels

	CTA	ED	GM	LO	LS	MN	MS	RV	SE	SB	OT
E[Pricing error]	0.10%	0.12%	0.19%	0.03%	-0.02%	0.04%	0.07%	0.12%	-0.06%	0.20%	0.09%
Skill	0.026	0.03	0.045	0.011	0.000	0.013	0.020	0.030	-0.009	0.048	0.024

4. Portfolio Methodology

In this section we describe the BL portfolio model we employ for our core analysis, together with the variance-based and other constraints which we impose, and the objective function we use.

4.1 Black-Litterman Portfolio Model

The Black-Litterman (BL) portfolio model is based on the idea of combining two sources of information: the investor's subjective estimates of mean asset returns and risks (also known as investor's 'views'), and the benchmark (or reference) portfolio used for the computation of the 'neutral' (also known as 'implied') returns, which are then combined with the investor's 'views'. The BL model has been widely used in the academic literature, see for instance Kolm *et al*. (2014), Bessler and Wolff (2015), Bessler *et al*. (2017), Platanakis and Sutcliffe (2017), Platanakis *et al*.

(2019a), Oikonomou *et al.* (2018) and Silva *et al*. (2017), and provides a robust and efficient way of dealing with estimation risk. Another advantage of the BL model is that investors can either stay neutral for certain assets, or provide return estimates if they feel confident in making forecasts. The BL model also allows investors to distinguish between weak and strong forecasts by incorporating a reliability figure for each forecast.

The computation of the implied returns used by the BL model is based on the portfolio weights of the reference portfolio. The column vector of the implied excess-returns $(\mathbf{H})$ used in the original Black and Litterman (1992) model is computed as follows:

$$\mathbf{H} = \lambda \mathbf{\Sigma} \mathbf{x}^{\text{Reference}}, \tag{3}$$

where λ denotes the investor's risk aversion coefficient, $\mathbf{\Sigma}$ is the sample covariance matrix, and $\mathbf{x}^{\text{Reference}}$ represents a column vector of the weights of the reference portfolio. The computation of $\mathbf{H}$ is based on a reverse-optimization process, under the assumption that the column vector $\mathbf{x}^{\text{Reference}}$ results from a mean-variance portfolio optimization procedure. The choice of the reference (or benchmark) portfolio $\left(\mathbf{x}^{\text{Reference}}\right)$ used for the computation of the column vector of implied returns $(\mathbf{H})$ influences the performance of the BL model. For this reason we follow Bessler *et al*. (2017) and Platanakis *et al*. (2019a), and employ two different benchmark portfolios: the *1/N* (equally weighted portfolio) and the minimum variance (MV) portfolio when short-selling is allowed. We denote the BL models estimated using the *1/N* and MV portfolios as the benchmark portfolio as BL(1/N) and BL(MV) respectively. The choice of the global minimum variance portfolio might be well suited for risk-averse investors, and the equally weighted portfolio for those who consider that return estimates have a high degree of parameter uncertainty.

The BL model combines the vector of implied excess-returns $(\mathbf{H})$ with the investor's subjective return estimates ('views') which are expressed in the column vector $\mathbf{Q}$. The reliability of each 'view' is also incorporated in the model. The column vector (posterior estimate) of mean returns $(\boldsymbol{\mu}_{\mathrm{BL}})$ is computed as follows:

$$\boldsymbol{\mu}_{\mathrm{BL}} = \left[(c\boldsymbol{\Sigma})^{-1} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1}\left[(c\boldsymbol{\Sigma})^{-1}\mathbf{H} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\mathbf{Q}\right]. \tag{4}$$

where $\boldsymbol{P}$ is a binary diagonal matrix with a value of unity if a subjective return estimate exists for the asset. The parameter c measures the reliability (the total level of confidence) in the vector of implied excess-returns $(\mathbf{H})$. As the parameter c tends to zero, the posterior estimates of the combined mean returns $(\boldsymbol{\mu}_{\mathrm{BL}})$ converge to the neutral (or implied) returns; and as c tends to infinity, the combined mean returns $(\boldsymbol{\mu}_{\mathrm{BL}})$ converge to the subjective return estimates ('views'). As in Platanakis and Sutcliffe (2017), we set the parameter c to 0.1625, which is the mean of the range of values used by previous studies. Bessler *et al*. (2017) show that the actual performance of the BL model is robust to the choice of c over the 0.025 to 1.00 range.

$\boldsymbol{\Omega}$ is a diagonal matrix quantifying the reliability measures for each asset on its diagonal, and is computed as follows (Meucci, 2010):

$$\boldsymbol{\Omega} = \frac{1}{\delta}\boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^{\mathbf{T}}. \tag{5}$$

The intuition behind the computation of the matrix $\boldsymbol{\Omega}$ is that the reliability of the investor's 'views' is a fraction of the corresponding reliability of implied ('neutral') returns with a factor $1/\delta$, which we set to unity (Meucci, 2010). We follow Bessler and Wolff (2015), Bessler *et al*.

(2017), Platanakis and Sutcliffe (2017) and Platanakis *et al*. (2019a) among others, and use the mean return for each asset during the corresponding estimation period as the investor's views.

Finally, Satchell and Scowcroft (2000), Bessler *et al*. (2017), Platanakis and Sutcliffe (2017) and Platanakis *et al*. (2019a), compute the posterior covariance matrix $\left(\boldsymbol{\Sigma}_{\mathrm{BL}}\right)$ as follows:-

$$\boldsymbol{\Sigma}_{\mathrm{BL}} = \boldsymbol{\Sigma} + \left[\left(c\boldsymbol{\Sigma}\right)^{-1} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Omega}^{-1}\boldsymbol{P}\right]^{-1}. \tag{6}$$

4.2 Variance-Based Constraints (VBCs)

For the core part of our analysis, we impose the variance-based constraints (VBCs) of Levy and Levy (2014) to further control for estimation risk. The great advantage of VBCs is that they constrain assets with higher estimation errors, as measured by their standard deviation, more strongly; and this may lead to better out-of-sample (actual) performance. The variance-based constraints (VBCs) are as follows:-

$$\left|\mathrm{x}_i - \frac{1}{N}\right| \frac{\sigma_i}{\bar{\sigma}} \leq \alpha, \quad \forall i \tag{7}$$

where $\bar{\sigma}$ denotes the mean standard deviation of returns across all the assets, σ_i is the standard deviation of the i^{th} asset, N is the number of assets, and x_i is the weight of the i^{th} asset. The effects of VBCs weaken as the parameter α increases, and when $\alpha = 0$ VBCs yield the naïve diversification strategy (*1/N*).

4.3 Portfolio Optimization

In the original Markowitz (1952) mean-variance portfolio model investors trade-off expected portfolio returns and the variance of portfolio returns. More than sixty years of research has generally been supportive of mean-variance portfolio analysis (Markowitz, 2014). We compute the optimal portfolio weights by maximizing the following quadratic utility function with respect to the portfolio weights (decision variables) using the BL estimates $(\boldsymbol{\mu}_{\mathrm{BL}}, \boldsymbol{\Sigma}_{\mathrm{BL}})$ for the mean returns and covariance matrix. Hence, the optimization process is described as follows:

$$\max_{\mathbf{x}} \mathrm{U} = \mathbf{x}^{\mathrm{T}}\boldsymbol{\mu}_{\mathrm{BL}} - \frac{\lambda}{2}\mathbf{x}^{\mathrm{T}}\boldsymbol{\Sigma}_{\mathrm{BL}}\mathbf{x}. \qquad (8)$$

where **x** is a column vector of asset weights. In addition, we impose non-short selling constraints $(\mathrm{x}_i \geq 0 \quad \forall i)$, normalization of portfolio weights $\left(\sum_{i=1}^{N} \mathrm{x}_i = 1\right)$.

5. Portfolio Results

In this section we present our empirical analysis. We use a 60 month rolling window approach to estimate the mean asset returns for the core part of our analysis. The mean expected returns next period (month, *t+1*) are estimated using returns for the preceding 60 months. The use of a rolling window allows forecasts of expected returns to be more responsive to structural breaks. The estimation of the covariance matrix uses an expanding window as this is expected to produce more stable estimates. At any point in time (month) $t \geq 60$, we use data available up to and including time (month) t to estimate the covariance matrix. Initially the estimation window is the first 60 months of our dataset. Then we use these asset weights to compute the out-of-sample (actual) portfolio return for the next period (month, *t+1*). We repeat this process by increasing the length our estimation window by one month until our dataset is exhausted.

Our portfolio models assume the covariance matrix is independent of the state of the market. When markets are down, e.g. a stock market crash, asset return correlations are higher than otherwise (Ang and Chen, 2002; Longin and Solnik, 2001). Chua et al (2009) and Page and Panariello (2018) show that hedge fund strategies have higher correlations with the stock market during down markets, and Billio et al (2017) find the correlation between hedge fund strategies increases by 33% during financial crises. For portfolio models that assume the correlation matrix is independent of the state of the market, this assumption leads to lower out-of-sample risk reduction benefits from diversification during down markets than expected. Therefore, due to under estimating asset correlations when the market is down, the out-of-sample performance of such portfolio models is reduced. Allowance for higher correlations during down markets should improve the out-of-sample performance of portfolios containing hedge funds, and so our empirical results tend to understate the benefits of hedge fund diversification. We investigate this in one of our robustness checks.

We use four performance metrics to assess the out-of-sample risk-adjusted performance of our portfolios: the Sharpe ratio (SR), MPPM, and the Omega and Sterling ratios. The SR is defined as follows:

$$SR = \frac{\bar{R}_p - \bar{R}_f}{\sigma_p}, \tag{9}$$

where $\bar{R}_p - \bar{R}_f$ denotes the mean excess portfolio return over the out-of-sample period, and σ_p represents the corresponding portfolio standard deviation over the same period. However, the SR has its limitations since it is based on only the first two moments. Smetters and Zhang (2014) show that the SR provides a correct and robust ranking when portfolio returns follow a normal distribution, but this may not be the case in the presence of non-normal returns. For this reason, we also use MPPM and the Omega and Sterling ratios as alternative performance metrics. The MPPM statistic $\hat{\Theta}$ of Goetzmann et al. (2007) is given by the following equation:

$$\hat{\Theta} \equiv \frac{1}{(1-\lambda)\Delta t} ln\left(\frac{1}{T}\sum_{t=1}^{T}\left[(1+r_t)/(1+rf_t) \right]^{1-\lambda} \right) \tag{10}$$

where Δ_t which is the length of time between observations, T which is the total number of observations, r_t and rf_t are the per period (not annualized) rate of return of the portfolio and the interest rate over period t, and λ is the investor's risk aversion coefficient. The Omega ratio (also known as a gain-loss ratio) is defined as the ratio of the average gain to the average loss, where the threshold target is set to zero for the computation of the gains/losses over the entire out-of-sample period (Shadwick and Keating, 2002). We compute the Sterling ratio as the mean excess portfolio return over the risk-free rate, divided by the mean drawdown rate over the entire out-of-sample period, as in Eling and Schuhmacher (2007).

Tables 5, 6 and 7 present the results for the BL model with λ = 2, 5 and 10 levels of risk aversion, representing aggressive, moderate and conservative investors, respectively.[14] In each of these tables we use four versions of the BL model with monthly rebalancing, a 60 month rolling window. Using the approach suggested by Jobson and Korkie (1980), as corrected by Memmel (2003), we examine the statistical significance of the differences between the SRs of returns on the benchmark portfolio (consisting of equities, bonds and the risk-free asset), and returns on each of the 11 portfolios that also contain one of the 11 HF strategies. Given two portfolios i and n, with as their estimated means, variances, and covariance over a sample of size $T-M$, the test of the hypothesis H_0 : $\hat{\mu}_i/\hat{\sigma}_i = \hat{\mu}_n/\hat{\sigma}_n$ is obtained by the test statistic Z, which is asymptotically distributed as a standard normal: $Z_{JK} = (\hat{\sigma}_n\hat{\mu}_i - \hat{\sigma}_i\hat{\mu}_n)/\sqrt{\vartheta}$ where $\vartheta = (2{\hat{\sigma}_i}^2{\hat{\sigma}_n}^2 - 2\hat{\sigma}_i\hat{\sigma}_n\hat{\sigma}_{i,n} + {\hat{\mu}_i}^2{\hat{\sigma}_n}^2/2 - \hat{\mu}_i\hat{\mu}_n{\hat{\sigma}_{i,n}}^2/\hat{\sigma}_i\hat{\sigma}_n)/(T-M)$.

[14] The underlying asset allocation tables are available on request. We do not provide them for the brevity reasons.

In Table 5 with $\lambda = 10$ all the strategies, except SB, have significantly higher SRs than the benchmark portfolio for all four versions of the BL model. For all the strategies, except ED and SB, the inclusion of a HF strategy also leads to an improvement in MPPM and the Omega and Sterling ratios in every case. Overall, of 176 performance measure comparisons in Table 5, only 4% of the benchmark values are higher than those of the corresponding portfolios which include a HF strategy, and all but one of these are due to SB.

[Insert Table 5 about here]

Table 6 presents the results for $\lambda = 5$. All the strategies, except MN, SE and SB, have significantly higher SRs than the benchmark portfolio for all four versions of the BL model. Out of 176 comparisons of the performance measures in Table 6, the benchmark score is higher just 5% of the time, and all but one of these are due to SB.

[Insert Table 6 about here]

Table 7 presents the results for $\lambda = 2$. The SRs for the CTA, ED, GM, MS, RV and OT strategies are significantly greater than for the benchmark portfolio across all four BL models; which is no longer the case for LO, LS, MN and SE. For only 7% of the 176 comparisons in Table 7 does the benchmark have a superior performance measure to the corresponding HF strategies, and half of these are due to SB.

[Insert Table 7 about here]

Overall, the CTA, ED, GM, MS, RV and OT strategies provide significant and consistent diversification benefits to investors, regardless of their level of risk aversion. This group comprises all four of the semi-directional strategies, and two of the three non-directional strategies, and none of the four directional strategies[15]. The LS and LO strategies also offer diversification benefits to less risk averse investors, the MN and SE strategies give a weaker improvement in performance; while the diversification benefits of the SB strategy are questionable. So diversification into every HF strategy, except SB, is beneficial. There is also evidence that HF strategies provide higher diversification benefits for more risk averse investors. For example, a comparison of Tables 5 and 7 shows that 60% of the performance measures for λ = 10 are larger than when λ = 2. If the results for BL(MV) with VBCs are ignored, this percentage rises to 75%. Guidolin and Orlov (2018) also find that investors with higher risk aversion benefit most from including hedge funds in a diversified portfolio, and that the increase in both Sharpe ratios and CERs when λ increases from 2 to 10 is statistically significant. In addition, Hagelin et al (2006) find that adding three Fund of Funds Composite index sub-indices to a US equity and bond portfolio leads to a larger increase in utility as risk aversion rises.

6. Robustness Checks

To check the robustness of our findings in Section 5 we repeat this analysis, but use different estimation windows, different portfolio models, a different utility function, and different data. In our first robustness check, instead of a 60 month rolling window, we use a 12-month expanding window for mean returns, while using a 12 month expanding window for estimating the

[15] See the Appendix for details of directional, semi-directional and non-directional strategies. The market portfolio is fully exposed to systematic risk, while most types of hedge fund have offsetting risks giving them a lower standard deviation of returns than equities. Semi and non-directional strategies have less downside risk, compared to the market portfolio and directional strategies (see Mamoghli and Daboussi, 2009; Perello, 2007) which is in alignment with our results for the CTA, ED, GM, MS, RV and OT strategies which provide significant and consistent diversification benefits.

covariance matrix. Similar to our core analysis, we use four versions of the BL model and three levels of risk aversion, λ = 2, 5, and 10. The results in Tables 8, 9 and 10 are little changed from the core results in Tables 5, 6 and 7. The CTA, ED, GM, LS, MS, RV and OT strategies have significantly larger SRs than the baseline portfolio for all four models and the three values of λ. These are the best six strategies in our core analysis, plus LS. Of the 176 comparisons in Tables 8, 9 and 10, the benchmark portfolio has a higher performance measure than the corresponding portfolio which includes a HF only 6%, 4% and 8% of the time. Comparing the performance measures for λ = 2 and 10 in Tables 8 and 10, the values in Table 8 are higher 60% of the time, indicating that HFs provide greater diversification benefits for risk averse investors. If the results for BL(MV) with VBCs are ignored, this percentage rises to 70%.

[Insert Table 8 about here]

[Insert Table 9 about here]

[Insert Table 10 about here]

Our second robustness test employs the Bayes–Stein portfolio model proposed by Jorion (1986), in conjunction with the VBCs of Levy and Levy (2014), and non-short selling constraints. We use a 12 month expanding window for estimating the means and covariance matrix, together with risk aversion parameters of λ = 2, 5, and 10. The Bayes-Stein model is based on the idea of shrinkage estimation, and is an alternative way of controlling for the negative effects of estimation errors in the optimization process. The Bayes-Stein model computes the column vector of mean returns $\left(\boldsymbol{\mu}_{BS}\right)$ as follows:

$$\boldsymbol{\mu}_{BS} = \left(1-g\right)\boldsymbol{\mu} + g\mu_G \mathbf{1}, \tag{9}$$

where the shrinkage factor $g\ (0 \le g \le 1)$ is computed as follows:

$$g = \frac{N+2}{(N+2) + T(\boldsymbol{\mu} - \mu_G \mathbf{1})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - \mu_G \mathbf{1})}. \tag{10}$$

μ_G is the expected return of the minimum variance portfolio when short sales are permitted, $\boldsymbol{\mu}$ is the column vector of historic returns, and T is the length of the estimation period. The Bayes–Stein estimator of the covariance matrix of asset returns $(\boldsymbol{\Sigma}_{BS})$ is given by:

$$\boldsymbol{\Sigma}_{BS} = \left(\frac{T+\varphi+1}{T+\varphi}\right)\boldsymbol{\Sigma} + \frac{\varphi}{T(T+\varphi+1)} \frac{\mathbf{1}\mathbf{1}^T}{\mathbf{1}^T \boldsymbol{\Sigma}^{-1} \mathbf{1}}, \tag{11}$$

where

$$\varphi = \frac{N+2}{(\boldsymbol{\mu} - \mu_G \mathbf{1})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - \mu_G \mathbf{1})}. \tag{12}$$

Table 11 shows that when Bayes-Stein is used the CTA, ED, GM, MN, MS, RV and OT strategies have significantly higher SRs than the benchmark across all three values of *λ*. Apart from the addition of MN, these are the best performers in our core analysis. Of the 132 values comparisons of the performance measures of the benchmark portfolio and the corresponding portfolio including a HF, the benchmark portfolio superior in only 9% of cases.

[Insert Table 11 about here]

HF returns have often been found to be non-normally distributed (Amin and Kat, 2003; Brooks and Kat, 2002, and Wegener *et al*., 2010), and our data is also non-normal. If returns are non-normal, portfolio optimization techniques that rely on the first two statistical moments $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

may suffer from weak out-of-sample performance (Cumming *et al.*, 2014). To examine this issue, our third robustness test includes higher moments in the utility function with different levels of risk aversion, λ = 2, 5, and 10; in conjunction with the BL model and VBCs. We use a CRRA (constant relative risk aversion) utility function, defined as follows:

$$U_{CRRA}(W) = \frac{1}{1-\lambda} W^{1-\lambda}, \quad \lambda > 0, \lambda \neq 1, \tag{13}$$

where W denotes the end-of-period wealth. The Taylor series expansion of the expected CRRA utility function up to the 4th order is expressed as follows:

$$E\left(U_{CRRA}(W)\right) \approx \frac{1}{1-\lambda}\bar{W}^{1-\lambda} - \frac{\lambda}{2}\bar{W}^{-(\lambda+1)}\sigma_p^2 + \frac{\lambda(\lambda+1)}{3!}\bar{W}^{-(\lambda+2)}s_p^3 - \frac{\lambda(\lambda+1)(\lambda+2)}{4!}\bar{W}^{-(\lambda+3)}k_p^4 \tag{14}$$

where $\bar{W} = 1 + \mu_p$. We use the BL estimates $(\boldsymbol{\mu}_{BL}, \boldsymbol{\Sigma}_{BL})$ together with *1/N* as the reference portfolio to compute μ_p (portfolio mean return), and σ_p^2 (portfolio variance), as well as the sample-based estimates of s_p^3 (skewness) and k_p^4 (kurtosis). We compute the optimal portfolio weights by maximizing the expected CRRA utility function, subject to VBCs, normalized portfolio weights, no short-selling, and a 12 month expanding window for both the means and covariance matrix.

Table 12 shows that the CTA, ED, GM, LS, MN, MS, RV and OT strategies have statistically higher SRs than the benchmark for all three values of λ, and this includes the six best strategies in our core analysis, and also LS and MN. Apart from the MPPM performance measure, all but one of the values for λ = 10 are higher than when λ = 2, indicating that the inclusion of HFs has greater diversification benefits for more risk averse investors.

[Insert Table 12 about here]

In our fourth robustness check we repeated our BL analysis for a 60 month expanding window, as well as for a 12 month rolling window, and the results are summarised in Table 13 (the full results appear in tables A1 to A6 of the online appendix). As a fifth robustness check we examined out-of-sample diversification performance using simulation. This has the advantage that the results are not driven by patterns in our time series data such as calendar effects, autocorrelation, regime shifts in the means, variances and correlations, fat tails, etc. We fitted a multivariate normal distribution to our data for the 1995-2014 period, and generated 1,000 observations for each HF strategy and asset class. We then used the four versions of the BL model with $\lambda = 2, 5$ and 10 and a 12 month expanding window to form portfolios. The simulation results are in tables A7 to A9 of the online appendix, and are summarised in Table 13.

Hedge fund returns are based on non-market valuations of the underlying assets, which can lead to smoothed returns. This induces or increases positive serial correlation in returns, and reduces the variance and correlations with other assets (e.g. Agarwal et al, 2011; Cassar and Gerakos, 2011; Bollen and Pool, 2008, 2009). In a sixth robustness check we controlled for this potential problem using the Geltner (1991, 1993) method which is simple and widely used[16]. The desmoothed return series (r_t°) is computed as:-

$$r_t^\circ = (r_t - \rho r_{t-1})/(1 - \rho)$$

where r_t is the original smoothed series, and ρ is its first order autocorrelation. We recomputed our base case results (60 month rolling window for BL(1/N), BL(MV), BL(1/N) with VBCs, and BL(MV) with VBCs; $\lambda = 2, 5$ and 10) using the desmoothed data, and the results appear in table

[16] The method of Geltner (1991, 1993) is popular for real estate returns, and has been applied to hedge funds by Bekkers et al (2009); Brooks and Kat (2002); and Hoevenaars et al (2008). Alternative methods for desmoothing hedge fund returns have been proposed by Getmansky et al (2004), Okunev and White (2003) and Pedersen et al (2004).

13, and tables A10 to A12 in the online appendix. When λ = 10, 162 of the 176 the performance measures for strategies which include a HF strategy are superior to those for the corresponding portfolio which excludes HFs. When λ = 5 the corresponding figure is 165 of 176 comparisons; and when λ = 2 it is 155 comparisons out of 176. For each value of λ, eight of the occasions when the addition of a HF did not improve performance were due to SB. Therefore, while there is a small reduction in the number of times the inclusion of a HF improves performance when we desmooth the data, our conclusion that HFs improve performance is unaffected.

Previous researchers (Racicot and Théoret, 2016, 2018, 2019; and Gregoriou et al., 2021) have found that HF strategies differ over the business cycle. To control for this, in our final robustness check we allow for the presence of two market regimes which have different asset means, variances and correlations. We applied the regimes methodology used by Platanakis et al (2019b) and used a Taylor series expansion for the CRRA utility function to incorporate higher moments in the portfolio construction process. We ruled out short sales and estimated a two-state multivariate regime switching model for each 60 month rolling window for λ = 2, 5 and 10. Our results appear in online appendix A13, and are summarised in Table 13. They show that only 9 of the 132 portfolios which include hedge funds are inferior to the corresponding benchmark portfolio, which is similar to the results in tables 5, 6 and 7. Therefore allowance for different regimes makes little difference to our conclusions regarding the diversification benefits of HF strategies.

In total we computed 81 portfolios for each of the 11 HF strategies, and the benchmark strategy, using different combinations of window length, portfolio model, risk aversion, rolling or expanding window, actual or simulated data and desmoothed data, i.e. 972 portfolios in total. The number of significant SRs for each type of hedge fund appear in Table 13, and in the final column is the total number of significant SRs out of 81 for each HF strategy. The ranking of the diversification performance of the HF strategies across all these robustness checks is consistent with our conclusion that the CTA, ED, GM, MS, RV and OT strategies are the best. LO, LS and MN also perform well as diversifiers, while SE is not very good, and SB shows little benefit. Like our core results, our robustness checks find support for the view that diversification into HFs is more valuable for more risk averse investors.

[Insert Table 13 about here]

7. Conclusions

We analyse the performance of 5,504 North-American hedge funds as individual investments and in portfolios. Using the stochastic discount factor approach of Li *et al.* (2016), we find that, of the 11 hedge fund strategies we consider, short bias and global macro have the best stand-alone performance, and only the long-short and sector strategies have a negative stand-alone performance. However, for most investors what matters is the benefit of adding hedge funds to a portfolio, and our study challenges previous results regarding the performance of hedge funds as portfolio diversifiers. Six hedge fund strategies provide out-of-sample diversification benefits in a consistent and significant way when added to our benchmark portfolio of equities and bonds. These six best diversifiers comprise all four of the semi-directional strategies (event driven, global macro, multi strategy and others) and two of the three non-directional hedge fund strategies (commodity trading advisor and relative value). These six strategies are amongst the best seven individual investments. The diversification benefits of the other five hedge fund strategies are lower and modest. Short bias provides no diversification benefit, despite being the best stand-alone strategy. The remaining four hedge fund strategies (market neutral, long only, long-short and sector) are neither good stand-alone investments; nor are they amongst the best six diversifiers. This group includes three of the four directional strategies (long only, long-short and sector). Our results indicate that, while there is generally a positive relationship between good stand-alone performance and diversification benefits, this is clearly not the case for the short bias strategy. We also find that the higher is the risk aversion of an investor, the more beneficial are hedge funds strategies as portfolio diversifiers, which suggests that hedge funds tend to reduce portfolio risk, rather than increase returns.

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Appendix – Hedge Fund Strategies

Commodity Trading Advisor (CTA). This strategy mostly trades futures or options contracts on behalf of investors. These HFs rely mainly on computerized systems, or on fundamental and technical analysis. (Non-directional)

Event Driven (ED). This driven strategy seeks to capitalize on opportunities arising during a company's lifecycle such as mergers and acquisitions, spin-offs, bankruptcies, restructuring, share buybacks, and re-capitalization. (Semi-directional)

Global Macro. (GM) This strategy seeks to exploit opportunistic directional investments in global markets, using almost all the available strategies and financial instruments. (Semi-directional)

Long Only (LO). This strategy invests mainly in equities using long positions, i.e. it is a buy-and-hold strategy, and has a high correlation with the market index. (Directional)

Long-Short (LS). This is one of the most common HF strategies. with long positions in under-priced stocks, and short positions in over-priced stocks. Most funds have a long net exposure, and a relatively high correlation with the market. (Directional)

Market Neutral (MN). This strategy aims to be unaffected by market movements, and has the objective of exploiting mispricings at the lowest possible risk. (Non-directional)

Multi Strategy (MS). This strategy may specialize in merger arbitrage, distressed securities or convertible bond and fixed income arbitrage. Some HFs also allocate part of their capital to the long-short strategy. It is closely related to the global macro strategy, but the latter is more directional. (Semi-directional)

Relative Value (RV). This strategy involves arbitrage transactions to profit from relative pricing anomalies between related instruments such as debt, equities, futures and options. The market neutral strategy can be considered a special case of this strategy. (Non-directional)

Sector (SE). In this strategy HF managers invest in a particular industry using fundamental analysis and their specialist knowledge. (Directional)

Short Bias (SB). This strategy involves a short net position in equities, with HF managers trying to profit from rare but extreme negative events. (Directional)

Others (OT). Strategies not covered above. It contains HFs that invest in 'PIPES' (private investment in public equity), start-ups, or are close-end funds. (Semi-directional)

Table 5: 60 months rolling estimation window for means, BL portfolio techniques, λ = 10

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=10. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 60 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=10 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.4911	0.9361***	0.9420***	0.8795***	0.7239***	0.7538***	0.6642**	1.0337***	1.0593***	0.6561*	0.5187	1.3605***
	MPPM (Θ%)	1.1364	2.8621	3.2888	2.6235	2.3453	2.4191	1.6613	3.6214	3.4340	1.9556	1.0973	4.8406
	Omega Ratio	1.9902	2.8360	2.6497	2.8005	2.1933	2.3075	2.4786	2.9360	3.0986	2.1707	2.7953	4.0486
	Sterling Ratio	0.1143	0.4050	0.2011	0.4462	0.1718	0.2158	0.2112	0.3386	0.3284	0.1477	0.2781	0.7048
BL (MV)	Sharpe Ratio	0.5804	0.9379***	1.2546***	1.0679***	0.9186***	0.9505**	0.8839**	1.4066***	1.8089***	0.8735***	0.4199	1.4390***
	MPPM (Θ%)	0.6449	1.1332	2.3771	1.3752	1.2552	1.3612	1.2870	3.1500	3.5158	1.1260	0.5222	3.5534
	Omega Ratio	4.8368	5.8816	4.9690	6.2468	5.2276	5.6809	5.2425	5.9113	7.7323	5.5509	3.9492	6.5914
	Sterling Ratio	0.4333	0.7911	0.3678	0.9778	0.5531	0.7351	0.7251	0.7782	1.0181	0.6699	0.3071	1.0424
BL (1/N) with VBCs	Sharpe Ratio	0.4936	0.9346***	0.8035***	0.8131***	0.6430**	0.6605**	0.6475**	0.8705***	0.9024***	0.6133*	0.5846	0.9798***
	MPPM (Θ%)	1.1557	2.9163	2.6647	2.4894	1.9142	2.0048	1.6832	2.9241	2.9414	1.7606	1.2545	3.3857
	Omega Ratio	1.9572	2.8016	2.3872	2.5689	2.0469	2.1118	2.3608	2.5560	2.6967	2.0592	2.9566	2.7974
	Sterling Ratio	0.1185	0.4035	0.1817	0.3667	0.1455	0.1746	0.2074	0.2715	0.2758	0.1327	0.2859	0.3676
BL (MV) with VBCs	Sharpe Ratio	0.3871	0.7658***	0.5920***	0.5734***	0.5766***	0.5673***	0.5364***	0.6642***	0.7105***	0.5507**	0.4728	0.6633***
	MPPM (Θ%)	0.7006	1.8283	1.4436	1.3062	1.5067	1.4115	1.1947	1.7173	1.8074	1.3726	0.7844	1.7283
	Omega Ratio	1.8601	2.7402	2.2659	2.3400	2.1137	2.1590	2.2719	2.3963	2.4989	2.1085	3.3134	2.4087
	Sterling Ratio	0.0805	0.3003	0.1440	0.1897	0.1440	0.1563	0.1637	0.1880	0.2122	0.1320	0.2219	0.2162

Table 6: 60 months rolling estimation window for means, BL portfolio techniques, λ = 5

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=5. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 60 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=5 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**										
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.4615	0.8928***	0.8928***	0.8636***	0.7069**	0.7485**	0.5428	1.0229***	0.9836***	0.6586*	0.4271	1.4245***
	MPPM (Θ%)	1.6050	3.4469	4.0140	3.2137	3.2966	3.2121	1.6314	4.3915	3.8394	2.8239	1.2214	5.7332
	Omega Ratio	1.9149	2.6542	2.5068	2.7500	2.1342	2.2752	2.2522	2.8556	2.9005	2.1653	2.3834	4.2202
	Sterling Ratio	0.0900	0.3524	0.1699	0.3936	0.1520	0.2074	0.1185	0.3158	0.2718	0.1310	0.1898	0.7417
BL (MV)	Sharpe Ratio	0.5567	0.9575***	1.2080***	1.0805***	0.9142***	0.9178**	0.8293**	1.3414***	1.6243***	0.8354**	0.4146	1.4779***
	MPPM (Θ%)	1.1471	2.0420	3.6511	2.4490	2.1608	2.3180	1.7800	4.3863	4.2512	1.9315	0.9203	4.9772
	Omega Ratio	2.8357	3.8296	3.8041	4.0858	3.3822	3.6018	3.5272	4.3921	5.9074	3.4216	2.5912	5.3633
	Sterling Ratio	0.2703	0.6489	0.2268	0.8558	0.3828	0.4973	0.4663	0.5290	0.6658	0.4205	0.2260	0.8876
BL (1/N) with VBCs	Sharpe Ratio	0.4935	0.9353***	0.7636***	0.7935***	0.6308**	0.6393**	0.6194**	0.8267***	0.8477***	0.5956	0.5993	0.9206***
	MPPM (Θ%)	1.8024	3.6241	3.3114	3.0750	2.8929	2.7348	2.0250	3.4891	3.4333	2.5705	1.5163	3.8361
	Omega Ratio	1.9266	2.7216	2.2787	2.4787	1.9994	2.0455	2.2740	2.4411	2.5581	2.0033	2.8515	2.6350
	Sterling Ratio	0.1072	0.3819	0.1658	0.3098	0.1323	0.1565	0.1669	0.2402	0.2441	0.1132	0.2847	0.3220
BL (MV) with VBCs	Sharpe Ratio	0.4912	0.9317***	0.7907***	0.7411***	0.6819***	0.6645***	0.6642***	0.8559***	0.9064***	0.6421**	0.5429	0.8899***
	MPPM (Θ%)	1.6458	2.7991	2.8260	2.2791	2.5369	2.2728	1.9641	3.1029	3.1514	2.2480	1.2048	3.2619
	Omega Ratio	1.9916	2.9966	2.4584	2.5825	2.2593	2.2768	2.4624	2.6085	2.7685	2.2376	2.9520	2.6890
	Sterling Ratio	0.1238	0.4231	0.1829	0.3089	0.1904	0.2048	0.2336	0.2545	0.2761	0.1780	0.2563	0.3069

Table 7: 60 months rolling estimation window for means, BL portfolio techniques, λ = 2

This table shows the results for four different versions of the Black-Litterman (BL) model and λ= 2. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 60 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=2 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.2195	0.7399***	0.7730***	0.7209***	0.5454***	0.6090**	0.2305	0.9406***	0.7657***	0.5009**	0.2331	1.4008***
	MPPM (Θ%)	0.9393	3.8169	4.3061	3.5428	3.4523	3.3952	0.8289	4.6907	3.6943	2.9173	0.7861	6.1346
	Omega Ratio	1.5517	2.2294	2.2610	2.3706	1.8374	1.9788	1.7054	2.6258	2.4250	1.8597	1.8380	4.0308
	Sterling Ratio	0.0245	0.2306	0.1322	0.1947	0.0783	0.1302	0.0258	0.2786	0.1617	0.0695	0.0528	0.7175
BL (MV)	Sharpe Ratio	0.4195	0.8333***	0.9661**	0.9572***	0.6717*	0.7114*	0.5070	1.1774***	1.2563***	0.6543*	0.2956	1.5074***
	MPPM (Θ%)	1.5234	3.3742	4.2562	3.6142	2.7769	3.0200	1.5090	4.9820	4.4509	2.6685	1.0499	5.9807
	Omega Ratio	1.9975	2.6414	2.8598	3.1058	2.3282	2.5124	2.4137	3.4732	3.9597	2.4194	1.8751	5.0108
	Sterling Ratio	0.1194	0.4099	0.1626	0.5689	0.1478	0.2736	0.1759	0.3939	0.4537	0.1949	0.0855	0.8680
BL (1/N) with VBCs	Sharpe Ratio	0.4796	0.9409***	0.7126***	0.7436***	0.5628	0.5694	0.5917**	0.7832***	0.8104***	0.5345	0.5849	0.8674***
	MPPM (Θ%)	2.2274	4.1640	3.6870	3.3555	3.3279	3.0275	2.2402	3.8444	3.7753	2.9738	1.5893	4.1386
	Omega Ratio	1.8789	2.6911	2.1642	2.3482	1.8694	1.9076	2.1950	2.3358	2.4470	1.8757	2.7662	2.4971
	Sterling Ratio	0.1015	0.3665	0.1440	0.2507	0.0965	0.1112	0.1416	0.2094	0.2207	0.0855	0.2876	0.2717
BL (MV) with VBCs	Sharpe Ratio	0.5324	0.9866***	0.7604***	0.8290***	0.6517**	0.6434*	0.6485**	0.8376***	0.8675***	0.6163	0.5766	0.9367***
	MPPM (Θ%)	2.2732	3.9786	3.5176	3.3769	3.1671	2.9176	2.2905	3.7922	3.7228	2.8687	1.5183	4.1686
	Omega Ratio	2.0139	2.8983	2.3155	2.5970	2.1004	2.1204	2.3621	2.4862	2.6194	2.1220	2.7453	2.7007
	Sterling Ratio	0.1319	0.4435	0.1633	0.3610	0.1528	0.1719	0.1923	0.2464	0.2504	0.1336	0.2502	0.3385

Table 8: 12 month expanding window estimation for means, BL portfolio techniques for λ = 10

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=10. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months expanding estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$.

λ=10 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.7002	1.0026***	0.9859***	0.9541***	0.8438**	0.9352***	0.8786***	1.1632***	1.0968***	0.8070*	0.8085	1.5251***
	MPPM (Θ%)	2.3331	3.9458	3.9222	3.7211	3.2386	3.8455	3.0432	4.8882	4.2852	3.0169	2.5163	6.8645
	Omega Ratio	2.3868	3.1372	2.9401	3.0142	2.4861	2.7511	3.0259	3.3679	3.3789	2.4934	3.4156	4.7582
	Sterling	0.2627	0.5997	0.3088	0.6241	0.2842	0.3932	0.4193	0.5160	0.4847	0.2767	0.0592	0.1093
BL (MV)	Sharpe Ratio	0.6461	0.8226***	1.1110***	0.9102***	0.8069**	0.9499***	1.0927***	1.4542***	1.4221***	0.7676**	0.7161	1.8692***
	MPPM (Θ%)	1.3769	1.8344	3.2853	2.1946	1.9023	2.4253	2.7405	4.4818	4.0618	1.7348	1.7187	6.4678
	Omega Ratio	4.6758	5.1818	4.8091	5.2678	4.4652	5.0366	5.5508	5.6332	5.9146	4.6209	4.4510	8.8043
	Sterling	0.8075	1.0811	0.4792	1.2081	0.6620	0.9067	1.1208	0.8632	0.7713	0.7302	0.7747	2.1942
BL (1/N) with VBCs	Sharpe Ratio	0.6723	1.0101***	0.9633***	0.9356***	0.8008**	0.8796***	0.8947***	1.0716***	1.0539***	0.7895*	0.7907	1.2438***
	MPPM (Θ%)	2.1162	3.7982	3.7515	3.5149	2.9390	3.4679	2.9667	4.2166	3.9683	2.8831	2.0988	5.1406
	Omega Ratio	2.3553	3.1456	2.8581	2.9100	2.4051	2.5937	3.0394	3.1591	3.2280	2.4482	3.5555	3.5974
	Sterling	0.2225	0.5733	0.3033	0.5434	0.2514	0.3219	0.4223	0.4518	0.4413	0.2506	0.4352	0.6507
BL (MV) with VBCs	Sharpe Ratio	0.5878	0.8905***	0.8045***	0.7837***	0.7505***	0.7982***	0.8535***	0.9359***	0.8986***	0.7326**	0.7467	1.0844***
	MPPM (Θ%)	1.6068	2.5113	2.4668	2.2655	2.3696	2.5112	2.5163	2.9967	2.8041	2.2505	1.6594	3.7740
	Omega Ratio	2.3408	3.4053	2.9162	2.9673	2.5898	2.7838	3.1331	3.1430	3.1443	2.6123	4.1683	3.5026
	Sterling	0.1691	0.4863	0.2914	0.3830	0.2566	0.3124	0.3952	0.3711	0.3506	0.2536	0.4717	0.5117

Table 9: 12 month expanding window estimation for means, BL portfolio techniques for $\lambda = 5$

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=5. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months expanding estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$.

λ=5 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.7021	0.9626***	0.9343**	0.9194**	0.8119*	0.8827**	0.7660	1.1088***	1.0147***	0.7836	0.7204	1.4580***
	MPPM (Θ%)	3.7334	5.2666	5.2472	4.8606	4.8261	5.1588	3.5529	6.2329	5.3251	4.5216	3.0117	8.0344
	Omega Ratio	2.3075	2.8695	2.7048	2.8345	2.3697	2.5924	2.6628	3.1246	3.0685	2.4073	2.9693	4.4731
	Sterling	0.2468	0.5271	0.2676	0.5757	0.2633	0.3537	0.2633	0.4683	0.4285	0.2464	0.3791	1.0275
BL (MV)	Sharpe Ratio	0.7322	0.8905***	1.1331**	0.9678**	0.8945**	1.0287***	0.9695**	1.3814***	1.3343***	0.8541*	0.7463	1.7537***
	MPPM (Θ%)	2.7928	3.4760	4.8625	3.8192	3.6866	4.4074	3.3634	6.1469	5.2854	3.4192	2.9205	7.9868
	Omega Ratio	3.2410	3.6854	3.8401	3.7784	3.3687	3.8563	4.2968	4.4287	4.8099	3.4137	3.2560	6.5708
	Sterling	0.6240	0.8277	0.4080	0.9162	0.5649	0.7598	0.9260	0.7555	0.7551	0.6343	0.5408	1.6331
BL (1/N) with VBCs	Sharpe Ratio	0.6715	1.0287***	0.9369***	0.9423***	0.7898*	0.8676***	0.8558***	1.0337***	1.0204***	0.7756	0.8026	1.1913***
	MPPM (Θ%)	3.0001	4.8309	4.7299	4.5087	4.3637	4.6772	3.4316	4.9820	4.7160	4.1749	2.4689	5.8292
	Omega Ratio	2.2863	3.0636	2.7393	2.8402	2.3277	2.5112	2.8673	3.0038	3.0773	2.3589	3.4409	3.3953
	Sterling	0.2053	0.5466	0.2883	0.5272	0.2327	0.3042	0.3208	0.4024	0.4084	0.2140	0.4382	0.5676
BL (MV) with VBCs	Sharpe Ratio	0.6691	0.9976***	0.9344***	0.8870***	0.8299***	0.8601***	0.9269***	1.0717***	1.0400***	0.8098**	0.7989	1.2544***
	MPPM (Θ%)	2.6578	3.7767	4.0893	3.5330	3.7133	3.7778	3.3817	4.7059	4.3665	3.5087	2.2690	5.6568
	Omega Ratio	2.4308	3.4279	2.9445	3.0297	2.6907	2.8014	3.2325	3.2389	3.3021	2.6983	3.6257	3.6762
	Sterling	0.2517	0.6396	0.3031	0.5310	0.3478	0.3897	0.4867	0.4542	0.4335	0.3621	0.4835	0.6646

Table 10: 12 Month expanding window estimation for means, BL portfolio techniques for λ = 2

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=2. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months expanding estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$.

λ=2 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.5939	0.8567***	0.8487***	0.8369**	0.7289*	0.8012**	0.5957	0.9432***	0.8513***	0.7002	0.5634	1.2282***
	MPPM (Θ%)	4.6187	6.4814	6.7277	6.1568	6.3067	6.5044	4.1184	6.9948	6.3436	5.7966	3.3660	8.4065
	Omega Ratio	2.0186	2.4336	2.4079	2.5460	2.0989	2.2586	2.1631	2.7286	2.5956	2.1274	2.4098	3.7640
	Sterling Return	0.1222	0.3220	0.2258	0.3236	0.1653	0.2395	0.1250	0.4145	0.2993	0.1519	0.1796	0.8532
BL (MV)	Sharpe Ratio	0.6711	0.8395**	0.9851**	0.8819**	0.7896*	0.8865*	0.6890	1.1352***	1.0421***	0.7791	0.6061	1.4499***
	MPPM (Θ%)	4.4974	5.7778	6.5311	5.5687	5.3682	6.0860	3.9258	7.3878	6.5319	5.3367	3.6436	8.8366
	Omega Ratio	2.4971	2.7706	2.8777	2.9513	2.5221	2.7759	2.9163	3.3629	3.3909	2.6363	2.5004	4.6690
	Sterling Return	0.3712	0.5147	0.2985	0.6419	0.3365	0.4445	0.3741	0.5575	0.5728	0.3793	0.2515	1.1298
BL (1/N) with VBCs	Sharpe Ratio	0.6679	1.0170***	0.8893***	0.8863***	0.7500	0.8072**	0.8192***	0.9813***	0.9772***	0.7373	0.8347	1.1148***
	MPPM (Θ%)	3.6911	5.5140	5.2870	4.9542	5.2682	5.2815	3.7556	5.4470	5.1856	5.0192	2.7354	6.1951
	Omega Ratio	2.2230	2.9573	2.5797	2.6328	2.2043	2.3351	2.6955	2.8174	2.9011	2.2285	3.4321	3.0953
	Sterling Return	0.1933	0.5015	0.2459	0.3799	0.1854	0.2221	0.2619	0.3351	0.3477	0.1676	0.4879	0.4434
BL (MV) with VBCs	Sharpe Ratio	0.7116	1.0715***	0.9666***	0.9533***	0.8026	0.8814**	0.8881***	1.0764***	1.0528***	0.8071	0.8300	1.2256***
	MPPM (Θ%)	3.5375	5.2057	5.3338	4.9267	4.8600	5.2705	3.7638	5.5606	5.2206	4.8272	2.7301	6.3543
	Omega Ratio	2.4306	3.2609	2.8411	2.9220	2.4423	2.5912	3.0281	3.1508	3.1976	2.5144	3.3638	3.5330
	Sterling Return	0.2599	0.6340	0.3148	0.5620	0.2654	0.3351	0.3702	0.4652	0.4495	0.2655	0.4839	0.6291

Table 11: Bayes-Stein 12 months expanding estimation window, λ =2, 5 and 10

This table shows the results of Bayes-Stein with VBCs, and λ=2, 5, and 10. We consider 12 months expanding estimation window with monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the underlying HF strategies. * denotes significance at p<0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

		Benchmark Portfolio (E+B+rf) plus each individual HF Style											
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
Bayes-Stein λ=2	Sharpe Ratio	0.6765	1.0052***	0.8496**	0.8565**	0.7112	0.7617	0.7779*	0.9387***	0.9375***	0.6999	0.8735*	1.0644***
	MPPM (Θ%)	3.9283	5.7124	5.3780	5.0192	5.2446	5.2272	3.7261	5.5611	5.3166	4.9836	3.0871	6.2995
	Omega Ratio	2.2033	2.8581	2.4443	2.5001	2.1051	2.2043	2.5162	2.6492	2.7360	2.1226	3.2131	2.8799
	Sterling Ratio	0.1962	0.4546	0.2107	0.3201	0.1523	0.1776	0.2082	0.2819	0.2946	0.1378	0.4036	0.3634
λ=5	Sharpe Ratio	0.6845	1.0361***	0.9026***	0.9104***	0.7769	0.8226**	0.8057**	0.9948***	0.9859***	0.7703	0.8642*	1.1273***
	MPPM (Θ%)	3.1955	5.0489	4.7040	4.4694	4.4233	4.5232	3.2796	4.9594	4.7267	4.2861	2.8167	5.7200
	Omega Ratio	2.2484	3.0043	2.6040	2.7143	2.2738	2.3857	2.6811	2.8475	2.9006	2.3277	3.2292	3.1256
	Sterling Ratio	0.1958	0.5263	0.2469	0.4091	0.1986	0.2303	0.2321	0.3340	0.3387	0.1877	0.4308	0.4532
λ=10	Sharpe Ratio	0.7243	1.0820***	0.9750***	0.9891***	0.8178	0.8911**	0.8740**	1.0646***	1.0605***	0.8029	0.8655	1.2052***
	MPPM (Θ%)	2.4177	4.2965	3.9297	3.9618	3.0499	3.6413	2.9125	4.3069	4.1151	2.9783	2.4677	5.1356
	Omega Ratio	2.4577	3.2761	2.8349	2.9976	2.4219	2.5747	2.9921	3.1021	3.1907	2.4686	3.3159	3.4778
	Sterling Ratio	0.2764	0.6508	0.3268	0.6059	0.2601	0.3352	0.3278	0.4546	0.4613	0.2509	0.4924	0.5953

Table 12: Higher moments - 12 months expanding estimation window, λ = 2, 5 and 10

This table shows the results of the Higher Moments with VBCs and λ= 2, 5, and 10. We consider 12 months expanding estimation window with monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the underlying HF strategies. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$.

		Benchmark Portfolio (E+B+rf) plus each individual HF Style											
Portfolio Construction Higher Moments	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
λ=2	Sharpe Ratio	0.6757	1.0058***	0.8773***	0.8775**	0.7387	0.7899*	0.7721*	0.9593***	0.9517***	0.7281	0.8739*	1.1015***
	MPPM (Θ%)	3.9005	5.5819	5.3426	4.9838	5.2582	5.2238	3.5833	5.4566	5.1918	5.0173	3.0780	6.2385
	Omega Ratio	2.2031	2.9000	2.5313	2.5901	2.1707	2.2917	2.5567	2.7573	2.8083	2.2004	3.2247	3.0353
	Sterling Ratio	0.1930	0.4668	0.2252	0.3526	0.1721	0.2005	0.1995	0.2980	0.3065	0.1563	0.4114	0.4049
λ=5	Sharpe Ratio	0.6827	1.0544***	0.9581***	0.9528***	0.7973*	0.8764***	0.8443**	1.0501***	1.0454***	0.7932	0.8597*	1.1950***
	MPPM (Θ%)	3.0568	5.0043	4.8864	4.6141	4.4299	4.7897	3.3211	5.0848	4.8499	4.3530	2.7786	5.8878
	Omega Ratio	2.3095	3.1203	2.7885	2.8855	2.3628	2.5334	2.8799	3.0537	3.1393	2.4122	3.2515	3.4343
	Sterling Ratio	0.2138	0.5890	0.3168	0.4944	0.2288	0.2993	0.2921	0.4298	0.4425	0.2176	0.4517	0.5723
λ=10	Sharpe Ratio	0.7479	1.0987***	0.9933***	0.9996***	0.8221	0.9117**	0.9219**	1.1090***	1.0991***	0.8158	0.8488	1.2630***
	MPPM (Θ%)	2.5296	4.2949	3.9415	3.9601	3.0320	3.6661	3.0861	4.4897	4.2642	3.0120	2.4027	5.3680
	Omega Ratio	2.5258	3.3729	2.9294	3.0388	2.5434	2.6951	3.1692	3.2457	3.3293	2.6050	3.3206	3.6667
	Sterling Ratio	0.3441	0.7249	0.3324	0.6576	0.3092	0.3763	0.4329	0.5047	0.4878	0.3522	0.5002	0.7164

Table 13: Numbers of Significant Sharpe Ratios for 81 Portfolios for Each Hedge Fund Strategy*

	12 months expanding window												12 months rolling window			60 months expanding window			60 months rolling window									Totals
	Black-Litterman 4 models			Bayes-Stein 1 model			Higher moments 1 model			1000 BL simulation 4 models			Black-Litterman 4 models			Black-Litterman 4 models			Black-Litterman 4 models			Desmoothed 4 models			Regimes 4 models			
λ	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10	2	5	10	
CTA	4	4	4	1	1	1	1	1	1	3	4	4	3	4	4	4	4	4	4	4	4	4	4	4	1	1	1	79
ED	4	4	4	1	1	1	1	1	1	4	4	4	4	4	4	4	4	4	4	4	4	1	3	4	1	1	1	77
GM	4	4	4	1	1	1	1	1	1	4	4	4	2	4	4	4	4	4	4	4	4	4	4	4	1	1	1	79
LO	2	4	4	0	0	0	0	1	1	4	4	4	3	4	4	3	4	4	3	4	4	1	2	2	1	0	0	63
LS	4	4	4	0	1	1	1	1	1	4	4	4	2	2	3	2	4	4	3	4	4	2	3	2	1	1	1	67
MN	2	0	4	1	1	1	1	1	1	4	4	4	2	3	4	1	3	4	2	3	4	1	3	3	0	0	0	57
MS	4	4	4	1	1	1	1	1	1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	1	1	1	81
RV	4	4	4	1	1	1	1	1	1	4	4	4	4	4	4	4	4	4	4	4	4	4	3	4	0	1	1	79
SE	0	0	4	0	0	0	0	0	0	3	4	4	0	2	2	1	2	4	2	3	4	2	1	2	1	1	0	42
SB	0	0	0	1	1	0	1	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	7
OT	4	4	4	1	1	1	1	1	1	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	1	1	1	81

*The detailed results for the BL 12 month rolling window, BKL 60 month expanding window, simulation models, desmoothed data and regimes are available in the online Appendix.

APPENDIX

Table A1: 12 months rolling estimation window for means, BL portfolio techniques, λ = 10

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=10. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=10 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**										
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.8958	1.1082**	1.3152***	1.1537***	1.0623**	1.0712*	1.0629**	1.3641***	1.4098***	0.9853	1.0179	1.6963***
	MPPM (Θ%)	3.2278	4.3034	5.2748	4.4730	4.4713	4.5522	3.6528	5.4057	5.1272	4.1232	3.1661	7.1901
	Omega Ratio	2.9315	3.5379	3.9414	3.6676	3.0028	3.2774	3.5877	4.2023	4.4954	3.0055	4.0596	6.0138
	Sterling Ratio	0.4763	0.8387	0.6105	1.0839	0.4423	0.5731	0.6904	0.7459	0.8553	0.4897	0.6479	1.4471
BL (MV)	Sharpe Ratio	0.8177	0.8982*	1.3786***	0.9601*	0.9615*	0.9187	1.0962***	1.4755***	1.8196***	0.8923	0.9026	1.7852***
	MPPM (Θ%)	1.8085	2.0491	3.8990	2.2976	2.3086	2.3431	2.6135	4.1819	4.4344	2.1482	2.2967	6.1426
	Omega Ratio	4.6194	4.6156	6.4914	4.9478	4.8146	4.7512	5.4520	7.0312	9.2098	4.6248	4.5073	10.5273
	Sterling Ratio	0.7819	0.8146	1.1360	0.9959	0.9510	0.8796	0.8674	1.7526	2.5013	0.8901	0.7210	3.0755
BL (1/N) with VBCs	Sharpe Ratio	0.7229	1.0712***	1.0677***	0.9651***	0.9079***	0.9253***	0.9166***	1.1084***	1.1380***	0.8684**	0.9937	1.2560***
	MPPM (Θ%)	2.3494	3.9905	4.0820	3.5106	3.5682	3.6079	3.0031	4.1519	4.1149	3.3154	2.5592	5.0150
	Omega Ratio	2.4400	3.3486	3.0930	3.0048	2.6076	2.7322	3.0442	3.2717	3.4269	2.6143	4.3607	3.6639
	Sterling Ratio	0.2637	0.6447	0.3824	0.5591	0.3107	0.3470	0.4194	0.4444	0.4846	0.3090	0.6922	0.6212
BL (MV) with VBCs	Sharpe Ratio	0.6160	0.9259***	0.9199***	0.8345***	0.7723***	0.8185***	0.8167***	0.9792***	1.0389***	0.7600**	0.9333*	1.1129***
	MPPM (Θ%)	1.7206	2.6323	2.9185	2.4105	2.4488	2.5677	2.3136	3.0336	3.2288	2.3470	2.1331	3.7902
	Omega Ratio	2.3905	3.4687	3.1540	3.0799	2.6473	2.8199	3.0765	3.3493	3.5083	2.6847	4.8072	3.6893
	Sterling Ratio	0.1820	0.4971	0.3569	0.3990	0.2527	0.3003	0.3244	0.4108	0.4681	0.2629	0.7133	0.5616

Table A2: 12 months rolling estimation window for means, BL portfolio techniques, λ = 5

This table shows the results for four different versions of the Black-Litterman (BL) model and λ= 5. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=5 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.9029	1.0804**	1.2201***	1.0888**	1.0420*	0.9931	0.9748	1.2534***	1.2198***	0.9488	0.9201	1.5559***
	MPPM (Θ%)	4.6528	5.7528	6.2513	5.5810	6.0509	5.8002	4.4242	6.3187	5.6538	5.7501	3.8147	7.7108
	Omega Ratio	2.8587	3.2038	3.6031	3.2826	2.9466	3.0393	3.0816	3.7547	3.7604	2.8810	3.2899	5.0067
	Sterling Ratio	0.5417	0.7329	0.5671	0.8921	0.4859	0.5457	0.6146	0.6871	0.7191	0.4656	0.4849	1.1058
Black-Litterman	Sharpe Ratio	0.8496	0.9400*	1.2745***	0.9985*	1.0018*	0.9398	0.9854*	1.3824***	1.4845***	0.8991	0.9485	1.7581***
	MPPM (Θ%)	3.1726	3.6367	5.1020	3.7690	3.9872	3.9927	3.4041	5.3756	5.0204	3.7328	3.7825	7.3305
	Omega Ratio	3.4627	3.4999	4.5203	3.7669	3.5974	3.6778	3.7241	4.9427	5.5743	3.4642	3.6126	7.5909
	Sterling Ratio	0.7113	0.7521	0.7140	0.8848	0.8306	0.7770	0.6320	1.1094	1.3402	0.7575	0.6595	1.9716
BL (1/N) with VBCs	Sharpe Ratio	0.7429	1.0861***	1.0423***	0.9735***	0.8911**	0.9139**	0.9187***	1.0904***	1.1118***	0.8590*	1.0066	1.2251***
	MPPM (Θ%)	3.2393	4.9206	4.8578	4.3766	4.6585	4.6027	3.5907	4.8641	4.7240	4.3784	2.9415	5.7342
	Omega Ratio	2.4215	3.2945	2.9799	2.9456	2.5215	2.6500	2.9883	3.1422	3.2785	2.5421	4.1991	3.4980
	Sterling Ratio	0.2683	0.6479	0.3631	0.5637	0.2978	0.3302	0.4180	0.4215	0.4545	0.2956	0.8472	0.5870
BL (MV) with VBCs	Sharpe Ratio	0.7017	1.0143***	1.0252***	0.8977***	0.8332**	0.8953***	0.8735***	1.0875***	1.1434***	0.8345*	1.0243*	1.2371***
	MPPM (Θ%)	2.7466	3.7965	4.2235	3.3940	3.6277	3.8184	3.1032	4.2638	4.3772	3.5832	2.7968	5.1668
	Omega Ratio	2.4922	3.4772	3.1840	3.0598	2.6609	2.8913	3.0693	3.4109	3.5950	2.7399	4.4350	3.8242
	Sterling Ratio	0.2443	0.5957	0.3884	0.4725	0.2976	0.3754	0.3623	0.4608	0.5421	0.3114	0.8935	0.6023

Table A3: 12 months rolling estimation window for means, BL portfolio techniques, $\lambda = 2$

This table shows the results for four different versions of the Black-Litterman (BL) model and λ= 2. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$.

λ=2 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.8186	0.9648*	1.1041**	0.9433	0.9733*	0.8941	0.8284	1.1012**	1.0024**	0.8997	0.6712	1.2843**
	MPPM (Θ%)	6.0245	7.1381	7.3687	6.6324	7.7602	6.9080	5.5346	7.4705	6.5293	7.5726	4.5307	8.0692
	Omega Ratio	2.4929	2.6928	3.1507	2.7638	2.7036	2.6302	2.5490	3.1591	3.0499	2.6015	2.3125	3.7389
	Sterling Ratio	0.3852	0.4446	0.4514	0.5014	0.3583	0.3522	0.3883	0.4890	0.5008	0.3245	0.1744	0.6794
BL (MV)	Sharpe Ratio	0.7973	0.9129	1.1164**	0.9071	0.9146	0.8443	0.7960	1.1166**	1.0702**	0.8457	0.7224	1.3951**
	MPPM (Θ%)	4.9787	5.9508	6.4487	5.4970	6.1922	5.6792	4.4666	6.5072	5.8190	6.2287	4.7647	7.6289
	Omega Ratio	2.7352	2.7615	3.3686	2.8843	2.7755	2.8160	2.6902	3.4274	3.4865	2.8056	2.4619	4.4783
	Sterling Ratio	0.4671	0.4340	0.4963	0.5442	0.4205	0.4198	0.3990	0.5497	0.7226	0.4287	0.2238	0.8348
BL (1/N) with VBCs	Sharpe Ratio	0.7665	1.0880**	1.0368**	0.9825**	0.8707*	0.8963*	0.9343**	1.0882**	1.1031**	0.8421	1.0179	1.2195**
	MPPM (Θ%)	3.9029	5.5351	5.4620	5.0113	5.4603	5.3142	4.0381	5.4135	5.1964	5.1521	3.2220	6.3066
	Omega Ratio	2.4322	3.2474	2.9233	2.9093	2.4425	2.5610	2.9863	3.0799	3.2057	2.4561	4.0708	3.4258
	Sterling Ratio	0.2796	0.6480	0.3518	0.5622	0.2744	0.3075	0.4228	0.4043	0.4437	0.2636	0.9384	0.5813
BL (MV) with VBCs	Sharpe Ratio	0.7148	1.0551**	1.0123**	0.9368**	0.8553*	0.8714*	0.9002**	1.0709**	1.1069**	0.8240	1.0384*	1.2105**
	MPPM (Θ%)	3.4408	4.9968	5.0338	4.4462	4.8777	4.7454	3.7454	5.0233	4.9570	4.5577	3.1957	5.9406
	Omega Ratio	2.4063	3.2541	2.9687	2.9295	2.5211	2.6497	2.9761	3.1565	3.3105	2.5513	4.1997	3.5055
	Sterling Ratio	0.2597	0.6181	0.3485	0.5279	0.2887	0.3225	0.3975	0.4122	0.4613	0.2971	0.9656	0.5714

Table A4: 60 months expanding estimation window for means, BL portfolio techniques, λ =10

This table shows the results for four different versions of the Black-Litterman (BL) model and λ = 10. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=10 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.5233	0.9469***	0.9261***	0.9002***	0.7281**	0.7693**	0.6988**	1.0142***	1.0415***	0.6671*	0.5672	1.3679***
	MPPM (Θ%)	1.2922	3.0948	3.3678	2.8597	2.3919	2.5851	1.8619	3.6980	3.5884	2.0494	1.2807	5.0340
	Omega Ratio	2.0061	2.8000	2.6136	2.7666	2.1856	2.2928	2.4779	2.8429	3.0505	2.1542	2.7614	3.9351
	Sterling	0.1430	0.4218	0.2261	0.4685	0.1917	0.2412	0.2351	0.3505	0.3658	0.1692	0.2743	0.6865
BL (MV)	Sharpe Ratio	0.7069	1.0428***	1.3049***	1.1886***	0.9825**	1.0354**	1.1148**	1.3817***	1.5424***	0.9322**	0.5549	1.7083***
	MPPM (Θ%)	0.8403	1.3564	2.9124	1.6886	1.5524	1.7314	1.8794	3.6647	3.7571	1.3594	0.7575	4.6568
	Omega Ratio	5.3171	6.1004	4.7898	6.4103	4.9188	5.1196	5.1371	5.0271	5.9661	5.3215	4.1052	6.8856
	Sterling	0.5967	0.9731	0.4803	1.2919	0.5759	0.7032	0.8711	0.6582	0.6980	0.6273	0.4215	1.3956
BL (1/N) with VBCs	Sharpe Ratio	0.5078	0.9431***	0.8261***	0.8283***	0.6672**	0.6830**	0.6884**	0.8893***	0.9199***	0.6404*	0.6285	1.0132***
	MPPM (Θ%)	1.2201	3.0881	2.8628	2.6343	2.0405	2.1620	1.8732	3.0887	3.1184	1.9131	1.4259	3.6028
	Omega Ratio	1.9722	2.7872	2.4178	2.5752	2.0772	2.1207	2.4141	2.5900	2.7334	2.0887	2.9037	2.8524
	Sterling	0.1303	0.4177	0.2062	0.3834	0.1666	0.1903	0.2444	0.2975	0.3112	0.1545	0.2927	0.4016
BL (MV) with VBCs	Sharpe Ratio	0.4050	0.7785***	0.6315***	0.6092***	0.6005***	0.5832***	0.6163***	0.7093***	0.6918***	0.5703***	0.5216	0.7478***
	MPPM (Θ%)	0.7718	1.8827	1.6013	1.4362	1.6121	1.4733	1.4750	1.9423	1.8304	1.4555	0.9033	2.0935
	Omega Ratio	1.8916	2.7815	2.3462	2.4209	2.1584	2.1987	2.4016	2.4480	2.4709	2.1561	3.2559	2.5354
	Sterling	0.0919	0.3194	0.1721	0.2268	0.1640	0.1726	0.2128	0.2173	0.2074	0.1542	0.2501	0.2565

Table A5: 60 months expanding estimation window for means, BL portfolio techniques, λ = 5

This table shows the results for four different versions of the Black-Litterman (BL) model and λ = 5. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=5 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.5232	0.9074***	0.8586**	0.8904***	0.7070**	0.7252*	0.5719	0.9759***	0.9655***	0.6484	0.4517	1.3997***
	MPPM (Θ%)	2.0655	3.8541	4.1788	3.5839	3.5312	3.3270	1.8843	4.5731	4.1589	2.9508	1.3129	5.9754
	Omega Ratio	1.9434	2.5824	2.4158	2.6546	2.1020	2.1658	2.1891	2.6716	2.8146	2.0895	2.3842	3.9764
	Sterling	0.1254	0.3602	0.1853	0.4381	0.1723	0.2079	0.1248	0.3087	0.3082	0.1419	0.1744	0.7004
BL (MV)	Sharpe Ratio	0.7293	1.0372***	1.2742***	1.1987***	1.0124**	1.0746**	1.0422*	1.3267**	1.5234***	0.9655**	0.5362	1.6372***
	MPPM (Θ%)	1.6397	2.4367	4.4378	3.0305	2.8853	3.1958	2.1996	4.9637	4.8011	2.5759	1.2964	5.8588
	Omega Ratio	3.1534	3.9136	3.8180	4.2656	3.4743	3.7084	4.2674	4.0106	5.2040	3.6257	2.7565	5.5321
	Sterling	0.4138	0.6716	0.3722	1.0167	0.4848	0.5988	0.7525	0.5897	0.7243	0.5278	0.2963	1.0917
BL (1/N) with VBCs	Sharpe Ratio	0.5056	0.9449***	0.7755***	0.8144***	0.6411**	0.6493**	0.6307**	0.8295***	0.8654***	0.6079	0.6543	0.9376***
	MPPM (Θ%)	1.9442	3.9054	3.5551	3.2963	3.1134	2.9385	2.1487	3.6324	3.6698	2.7620	1.7428	4.0218
	Omega Ratio	1.9176	2.6864	2.2897	2.4859	1.9952	2.0301	2.2537	2.4319	2.5745	1.9903	2.8402	2.6549
	Sterling	0.1208	0.3951	0.1892	0.3627	0.1498	0.1727	0.1759	0.2550	0.2792	0.1263	0.2968	0.3372
BL (MV) with VBCs	Sharpe Ratio	0.5299	0.9632***	0.8001***	0.7957***	0.7350***	0.6901***	0.7486***	0.8956***	0.9077**	0.7044***	0.6271	1.0258***
	MPPM (Θ%)	1.8399	3.0205	3.0688	2.5869	2.8391	2.4823	2.3109	3.5622	3.4622	2.5726	1.5088	4.0725
	Omega Ratio	2.0578	3.0803	2.4678	2.6859	2.3662	2.3114	2.5997	2.6544	2.7794	2.3581	2.9393	2.9106
	Sterling	0.1562	0.4691	0.1982	0.3706	0.2429	0.2330	0.2957	0.2990	0.3024	0.2418	0.3043	0.4112

Table A6: 60 months expanding estimation window for means, BL portfolio techniques, $\lambda = 2$

This table shows the results for four different versions of the Black-Litterman (BL) model and $\lambda = 2$. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$.

λ=2 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.3185	0.7567***	0.7345***	0.7453**	0.5637**	0.6050**	0.3348	0.8839***	0.7701***	0.5051*	0.2548	1.3496***
	MPPM (Θ%)	1.6839	4.4395	4.7046	4.0275	4.0647	3.9091	1.4414	4.7805	4.1420	3.2501	0.8815	6.1412
	Omega Ratio	1.5916	2.1409	2.1242	2.2895	1.8012	1.8635	1.7149	2.4525	2.3453	1.7793	1.8801	3.8137
	Sterling	0.0410	0.2022	0.1377	0.1968	0.0928	0.1272	0.0390	0.2695	0.1798	0.0741	0.0451	0.6433
BL (MV)	Sharpe Ratio	0.5615	0.8809**	0.9870**	0.9827**	0.7955*	0.7967	0.5883	1.1689**	1.2064***	0.7715	0.3435*	1.5807***
	MPPM (Θ%)	2.2088	3.8201	5.1873	4.0449	3.8462	3.9416	1.6902	5.6458	5.0224	3.5732	1.2132	6.7509
	Omega Ratio	2.2369	2.6849	2.7113	3.0639	2.4018	2.4187	2.7346	3.2056	3.6358	2.5489	2.0242	4.7741
	Sterling	0.1978	0.3869	0.2127	0.6017	0.2401	0.2861	0.1707	0.4182	0.4830	0.2611	0.0817	0.8836
BL (1/N) with VBCs	Sharpe Ratio	0.5068	0.9184***	0.7122***	0.7295***	0.5807	0.5701	0.5833	0.7666***	0.8057***	0.5441	0.6812	0.8430***
	MPPM (Θ%)	2.5921	4.4315	3.9716	3.5345	3.7339	3.2848	2.3782	4.0064	4.0371	3.2511	1.9179	4.2602
	Omega Ratio	1.8708	2.5624	2.1354	2.2534	1.8636	1.8695	2.1073	2.2677	2.4004	1.8485	2.8299	2.3983
	Sterling	0.1168	0.3561	0.1570	0.2433	0.1137	0.1191	0.1404	0.2080	0.2318	0.0933	0.3208	0.2559
BL (MV) with VBCs	Sharpe Ratio	0.5496	1.0160***	0.8120***	0.8449***	0.6630*	0.6741*	0.6643*	0.8824***	0.9043***	0.6669	0.6796	0.9824***
	MPPM (Θ%)	2.4158	4.2516	4.0944	3.6237	3.4828	3.4218	2.4005	4.1863	4.1282	3.3468	1.9209	4.4917
	Omega Ratio	2.0475	2.9218	2.3832	2.5963	2.1009	2.1059	2.3801	2.5675	2.6866	2.1618	2.7940	2.7798
	Sterling	0.1554	0.4741	0.2081	0.3960	0.1693	0.1922	0.2032	0.3006	0.3105	0.1629	0.3165	0.3792

Table A7: Simulated results – normal distribution (1000 observations), 12 months expanding estimation window for means, λ = 10

This table shows the results for four different versions of the Black-Litterman (BL) model with simulations, and λ = 10. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$

λ=10 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.7333	1.0141***	1.1220***	0.9690***	0.9390***	1.0335***	1.0014***	1.3074***	1.2936***	0.8372**	0.7865	1.7179***
	MPPM (Θ%)	2.3607	3.7184	4.3564	3.5538	3.8452	4.2742	3.1338	5.1610	4.7146	3.1971	1.9505	6.9725
	Omega Ratio	2.6696	3.2274	3.5279	3.1282	2.8472	3.0895	3.6007	4.0816	4.2084	2.6462	3.4768	5.7165
	Sterling	0.2947	0.5024	0.5588	0.4807	0.3588	0.4299	0.5971	0.7155	0.7968	0.2925	0.4084	1.5754
BL (MV)	Sharpe Ratio	0.7078	0.9450**	1.2312***	0.9872***	0.9659**	1.0458***	1.2283***	1.4793***	1.5782***	0.8513**	0.7645	1.9865***
	MPPM (Θ%)	0.8234	1.4243	2.1301	1.5997	1.4321	1.7504	2.1441	3.1267	3.3409	1.2374	1.0228	5.0203
	Omega Ratio	8.6650	7.3151	7.3561	6.9466	7.2767	6.6762	7.8176	7.2885	8.0671	6.9977	7.3840	9.9206
	Sterling	1.1217	1.2412	1.5190	1.3024	1.2273	1.1864	1.8389	1.5659	1.9464	1.0325	1.0146	3.3466
BL (1/N) with VBCs	Sharpe Ratio	0.7207	1.0094***	0.9964***	0.9410***	0.8633***	0.9248***	0.9704***	1.1238***	1.1094***	0.8110***	0.7793	1.3189***
	MPPM (Θ%)	2.3388	3.7060	3.8238	3.4390	3.4175	3.6776	3.1148	4.3290	4.0703	3.0637	1.9548	5.2262
	Omega Ratio	2.6160	3.2144	3.1631	3.0547	2.6817	2.8533	3.4438	3.5279	3.5840	2.5965	3.4196	4.1195
	Sterling	0.2821	0.5009	0.4569	0.4495	0.3185	0.3682	0.5447	0.5690	0.5987	0.2823	0.3949	0.9085
BL (MV) with VBCs	Sharpe Ratio	0.6415	0.8919***	0.8026***	0.7905***	0.7829***	0.7971***	0.8732***	0.8430***	0.8333***	0.7409***	0.7143	0.9606***
	MPPM (Θ%)	1.8436	2.4042	2.2210	2.1521	2.4836	2.3790	2.4175	2.3459	2.2914	2.2581	1.4208	2.8183
	Omega Ratio	2.6485	3.6435	3.3189	3.2963	2.8981	3.0778	3.5286	3.4253	3.4249	2.8439	4.0788	3.6636
	Sterling	0.2568	0.5387	0.4222	0.4253	0.3306	0.3730	0.5057	0.4551	0.4515	0.3072	0.4397	0.5807

Table A8: Simulated results – normal distribution (1000 observations), 12 months expanding estimation window for means, λ = 5

This table shows the results for four different versions of the Black-Litterman (BL) model with simulations, and λ =5. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01

λ=5 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.7531	1.0167**	1.1279***	0.9785***	0.9516***	1.0314***	0.9633***	1.3314***	1.3115***	0.8417**	0.8313	1.7874***
	MPPM (Θ%)	3.2936	4.9231	5.4872	4.6869	5.3068	5.6278	3.5862	6.4375	5.6557	4.4607	2.4930	8.3193
	Omega Ratio	2.5693	3.0080	3.3940	2.9735	2.7636	2.9485	3.3583	3.9579	4.1163	2.5389	3.3590	5.8552
	Sterling	0.2877	0.4509	0.5137	0.4540	0.3386	0.3887	0.5227	0.6702	0.7726	0.2707	0.4370	1.6346
BL (MV)	Sharpe Ratio	0.7711	1.0027*	1.2790***	1.0314***	1.0153**	1.1047***	1.2621***	1.5212***	1.6031***	0.9037**	0.8107	2.0137***
	MPPM (Θ%)	1.6004	2.7660	3.9158	3.0188	2.7495	3.3353	2.9271	5.3810	5.1761	2.3538	1.7670	7.6832
	Omega Ratio	4.4312	4.2121	4.7426	4.1625	4.2114	4.2280	6.0346	5.2588	5.9499	4.0127	4.3446	7.7640
	Sterling	0.5956	0.6873	0.9303	0.7513	0.6896	0.7117	1.4680	1.0800	1.4270	0.5609	0.6016	2.5115
BL (1/N) with VBCs	Sharpe Ratio	0.7260	1.0154***	1.0025***	0.9422***	0.8671***	0.9303***	0.9420***	1.1078***	1.0885***	0.8156***	0.8206	1.2883***
	MPPM (Θ%)	3.1546	4.7330	4.7608	4.3431	4.6442	4.7788	3.5880	5.0790	4.7319	4.2241	2.4791	5.8902
	Omega Ratio	2.5220	3.0695	3.0899	2.9395	2.6062	2.7827	3.2575	3.4016	3.4357	2.5139	3.3179	3.9302
	Sterling	0.2692	0.4713	0.4440	0.4259	0.3058	0.3581	0.4888	0.5379	0.5589	0.2695	0.4170	0.8374
BL (MV) with VBCs	Sharpe Ratio	0.7076	0.9905***	0.9189***	0.8977***	0.8564***	0.8761***	0.9975***	1.0807***	1.0845***	0.8166***	0.7919	1.2988***
	MPPM (Θ%)	2.6449	3.4830	3.3537	3.2376	3.3803	3.3187	3.3734	4.2991	4.1959	3.1205	2.0540	5.4192
	Omega Ratio	2.7032	3.5213	3.2976	3.2550	2.9733	3.0881	3.6708	3.5756	3.6441	2.9165	3.6452	4.1553
	Sterling	0.2902	0.5606	0.4631	0.4700	0.3819	0.4109	0.6087	0.5651	0.5991	0.3490	0.4552	0.9040

Table A9: Simulated results – normal distribution (1000 observations), 12 months expanding estimation window for means, λ = 2

This table shows the results for four different versions of the Black-Litterman (BL) model with simulations, and λ = 2. BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider 12 months rolling estimation window – monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$

λ=2 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**										
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.7062	0.9788**	1.1035***	0.9474***	0.8922**	0.9826***	0.8452*	1.3243***	1.2715***	0.7921*	0.8214	1.7939***
	MPPM (Θ%)	4.0762	6.6016	7.0207	5.9271	7.1394	7.4953	4.1021	7.2436	6.2666	6.0226	3.1063	8.7522
	Omega Ratio	2.3345	2.6696	3.0928	2.7043	2.4669	2.6487	2.7968	3.8252	3.8511	2.2789	3.0541	5.8597
	Sterling	0.2291	0.3616	0.4371	0.3748	0.2538	0.3154	0.3525	0.6416	0.6900	0.2093	0.3518	1.6317
BL (MV)	Sharpe Ratio	0.8049	1.0028	1.2093**	1.0133**	0.9968*	1.0561**	1.1194*	1.4608***	1.5492***	0.8859	0.8203	1.9902***
	MPPM (Θ%)	2.9433	5.2921	5.9921	5.0767	5.1361	5.8968	3.3606	7.2320	6.1039	4.4388	2.9153	8.9764
	Omega Ratio	3.0442	2.9824	3.6710	3.0991	3.0790	3.1312	4.4665	4.3798	5.2203	2.8309	3.1544	7.0373
	Sterling	0.3684	0.4321	0.5847	0.4790	0.4104	0.4215	0.8834	0.8145	1.1651	0.3297	0.3690	2.1809
BL (1/N) with VBCs	Sharpe Ratio	0.7224	1.0032***	0.9433***	0.9097***	0.8310***	0.8831***	0.9033***	1.0471***	1.0320***	0.7822**	0.8383	1.2059***
	MPPM (Θ%)	3.9400	5.4989	5.4244	5.0265	5.6269	5.5932	3.9946	5.6749	5.2837	5.1595	2.8878	6.4408
	Omega Ratio	2.4097	2.9432	2.8395	2.7699	2.4515	2.5931	3.0432	3.1270	3.1652	2.3644	3.2227	3.5471
	Sterling	0.2494	0.4393	0.3767	0.3814	0.2605	0.3082	0.4310	0.4638	0.4859	0.2366	0.4299	0.6880
BL (MV) with VBCs	Sharpe Ratio	0.7539	1.0363***	1.0134***	0.9648***	0.8849***	0.9387***	0.9839***	1.1445***	1.1172***	0.8330**	0.8389	1.3359***
	MPPM (Θ%)	3.3674	4.9553	5.0174	4.5984	4.7716	5.0430	3.7412	5.4029	4.9937	4.3709	2.7926	6.2145
	Omega Ratio	2.6853	3.2010	3.1748	3.0639	2.7598	2.8718	3.4991	3.5609	3.5824	2.6688	3.2903	4.1633
	Sterling	0.3030	0.5081	0.4671	0.4618	0.3398	0.3768	0.5603	0.5855	0.6007	0.2986	0.4414	0.9249

Table A10: 60 months rolling estimation window for means, BL portfolio techniques, λ = 10 (desmoothed HF data)

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=10 for desmoothed HF data following Geltner (1991, 1993). BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider a 60 month rolling estimation window, with monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=10 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.4911	0.9249***	0.6563*	0.8676***	0.5547	0.6130	0.6407*	0.7820**	0.7094**	0.5583	0.4617	1.1507***
	MPPM (Θ%)	1.9393	3.4509	3.2568	3.2058	3.0122	2.8851	2.1224	3.5957	3.0514	2.6951	1.4214	4.9983
	Omega Ratio	1.9902	2.8151	2.0849	2.7661	1.8956	2.0371	2.4162	2.3688	2.3361	1.9755	2.6444	3.3346
	Sterling	0.1143	0.3985	0.1750	0.4368	0.1387	0.1762	0.2018	0.2616	0.2283	0.1223	0.2406	0.6201
BL (MV)	Sharpe Ratio	0.5804	0.9151***	0.8346**	1.0477***	0.7129**	0.8524**	0.7781	0.9648**	0.8902***	0.7680**	0.3809	1.1746***
	MPPM (Θ%)	0.7038	1.1722	1.2412	1.4179	0.9108	1.1427	1.1744	1.6907	1.2721	0.9651	0.5760	2.5185
	Omega Ratio	4.8368	5.7553	4.5480	6.1521	5.0549	5.6788	4.8779	4.9492	5.1264	5.3257	3.7932	5.5867
	Sterling	0.4333	0.7277	0.4771	0.9526	0.4885	0.7501	0.6294	0.6038	0.5594	0.5671	0.2822	1.0066
BL (1/N) with VBCs	Sharpe Ratio	0.4936	0.9246***	0.6260*	0.8001***	0.5366	0.5806	0.6328**	0.7142**	0.7003***	0.5481	0.5443	0.8787***
	MPPM (Θ%)	2.0752	3.5518	3.1403	3.1805	2.9565	2.8397	2.2295	3.3725	3.1296	2.7294	1.5720	3.9396
	Omega Ratio	1.9572	2.7797	2.0320	2.5397	1.8596	1.9608	2.3247	2.2183	2.2904	1.9366	2.8325	2.5570
	Sterling	0.1185	0.4011	0.1630	0.3564	0.1310	0.1590	0.2034	0.2207	0.2259	0.1181	0.2683	0.3346
BL (MV) with VBCs	Sharpe Ratio	0.3871	0.7637***	0.5563***	0.5745***	0.5008**	0.5176**	0.5089***	0.5652***	0.5535***	0.5070*	0.4187	0.5955***
	MPPM (Θ%)	1.4566	2.1840	2.1059	1.7420	2.2495	1.9850	1.5366	1.8973	1.8228	2.0332	0.9371	1.9477
	Omega Ratio	1.8601	2.7361	2.1185	2.3417	1.9190	2.0177	2.2255	2.2131	2.2541	1.9903	3.0441	2.2909
	Sterling	0.0805	0.3029	0.1577	0.1901	0.1303	0.1427	0.1556	0.1690	0.1773	0.1227	0.1905	0.1843

Table A11: 60 months rolling estimation window for means, BL portfolio techniques, λ = 5 (desmoothed HF data)

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=5 for desmoothed HF data following Geltner (1991, 1993). BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider a 60 month rolling estimation window, with monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

λ=5 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**											
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.4615	0.8841***	0.6261	0.8485***	0.5438	0.6185*	0.5263	0.7420**	0.6595**	0.5536	0.3759	1.1286***
	MPPM (Θ%)	1.9608	3.7153	3.3785	3.4379	3.1219	3.1045	1.8286	3.8029	3.0706	2.8321	1.3209	5.4215
	Omega Ratio	1.9149	2.6419	1.9918	2.7101	1.8605	2.0247	2.1948	2.2507	2.2017	1.9613	2.2820	3.2354
	Sterling Ratio	0.0900	0.3486	0.1517	0.3810	0.1208	0.1722	0.1113	0.2305	0.1916	0.1097	0.1760	0.6152
BL (MV)	Sharpe Ratio	0.5567	0.9254***	0.8321**	1.0604***	0.7366**	0.8322**	0.7606*	0.9822**	0.9134***	0.7409**	0.3524	1.1733***
	MPPM (Θ%)	1.2297	2.0510	2.1312	2.4725	1.6755	1.9684	1.8110	3.0042	2.2469	1.6606	0.9078	4.1875
	Omega Ratio	2.8357	3.7214	3.0751	4.0291	3.1672	3.4604	3.2921	3.4379	3.4810	3.2230	2.4617	4.1760
	Sterling Ratio	0.2703	0.5906	0.3337	0.8050	0.3894	0.4772	0.4302	0.5135	0.4733	0.3667	0.1922	0.7758
BL (1/N) with VBCs	Sharpe Ratio	0.4935	0.9345***	0.6181*	0.7854***	0.5307	0.5615	0.6037*	0.7002**	0.6714**	0.5418	0.5406	0.8477***
	MPPM (Θ%)	2.1839	3.9204	3.3204	3.3597	3.1060	2.9334	2.2350	3.5302	3.2315	2.8987	1.6112	4.0157
	Omega Ratio	1.9266	2.7262	1.9836	2.4593	1.8276	1.9027	2.2328	2.1575	2.1875	1.9014	2.6736	2.4692
	Sterling Ratio	0.1072	0.3823	0.1536	0.3116	0.1178	0.1395	0.1588	0.2076	0.2018	0.1056	0.2573	0.3061
BL (MV) with VBCs	Sharpe Ratio	0.4912	0.9126***	0.6520**	0.7333***	0.5914*	0.6184**	0.6437**	0.7038***	0.6750***	0.5963	0.5045	0.7586***
	MPPM (Θ%)	1.9413	2.9082	2.5941	2.4399	2.7577	2.4713	2.0884	2.7042	2.4150	2.5066	1.3016	2.9564
	Omega Ratio	1.9916	2.9618	2.2377	2.5727	2.0433	2.1554	2.4120	2.3507	2.4083	2.1126	2.7928	2.4462
	Sterling Ratio	0.1238	0.4038	0.1922	0.3050	0.1752	0.1938	0.2248	0.2345	0.2324	0.1699	0.2278	0.2723

Table A12: 60 months rolling estimation window for means, BL portfolio techniques, λ = 2 (desmoothed HF data)

This table shows the results for four different versions of the Black-Litterman (BL) model and λ=2 for desmoothed HF data following Geltner (1991, 1993). BL(1 over N); BL(MV); BL(1 over N)-VBCs; BL(MV)-VBCs. We consider a 60 month rolling estimation window, with monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at $p < 0.1$, ** denotes significance at $p < 0.05$ and *** denotes significance at $p < 0.01$.

λ=2 (risk tolerance)		**Benchmark Portfolio (E+B+rf) plus each individual HF Style**										
Portfolio Construction Method	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
BL (1/N)	Sharpe Ratio	0.2195	0.7110***	0.4480*	0.7373***	0.3830*	0.4717**	0.2072	0.6403**	0.4762**	0.3911*	0.2075	1.1439**
	MPPM (Θ%)	0.9403	3.7447	2.8440	3.7356	2.3852	2.7242	0.7389	3.8885	2.6305	2.2396	0.7201	5.9217
	Omega Ratio	1.5517	2.1727	1.6819	2.3805	1.6194	1.7710	1.6568	1.9955	1.8506	1.6903	1.7506	3.1765
	Sterling	0.0245	0.2154	0.0739	0.2056	0.0578	0.0933	0.0222	0.1835	0.0924	0.0542	0.0489	0.6133
BL (MV)	Sharpe Ratio	0.4195	0.7968**	0.5875	0.9484***	0.5378	0.6720*	0.4468	0.7463*	0.6873**	0.5756*	0.2817	1.1068***
	MPPM (Θ%)	1.5234	3.2612	2.5748	3.6976	2.0608	2.6424	1.4198	3.6632	2.7517	2.1421	1.0634	5.2477
	Omega Ratio	1.9975	2.5778	2.1245	3.0277	2.1629	2.4321	2.2097	2.3975	2.4496	2.2540	1.8373	3.4004
	Sterling	0.1194	0.3925	0.1286	0.5466	0.1590	0.3069	0.1334	0.2634	0.2403	0.1792	0.0805	0.6019
BL (1/N) with VBCs	Sharpe Ratio	0.4796	0.9344***	0.5675	0.7374***	0.4785	0.5006	0.5723	0.6523**	0.6367**	0.4822	0.5275	0.7782***
	MPPM (Θ%)	2.2274	4.1560	3.2412	3.3381	2.9970	2.7847	2.1962	3.4687	3.2650	2.7891	1.5540	3.8817
	Omega Ratio	1.8789	2.6782	1.8818	2.3326	1.7240	1.7846	2.1468	2.0579	2.0966	1.7800	2.6016	2.3038
	Sterling	0.1015	0.3659	0.1245	0.2479	0.0882	0.0995	0.1330	0.1682	0.1746	0.0786	0.2569	0.2387
BL (MV) with VBCs	Sharpe Ratio	0.5324	0.9815***	0.6434	0.8217***	0.6020	0.5928	0.6329*	0.7021*	0.7007**	0.6080	0.5354	0.8443***
	MPPM (Θ%)	2.2732	3.9679	3.0504	3.3607	3.0484	2.7496	2.2619	3.3229	3.1060	2.8432	1.5177	3.8818
	Omega Ratio	2.0139	2.8852	2.0884	2.5739	2.0182	2.0330	2.3253	2.1969	2.2807	2.0955	2.6123	2.4957
	Sterling	0.1319	0.4394	0.1665	0.3513	0.1674	0.1758	0.1821	0.2107	0.2216	0.1494	0.2198	0.3035

Table A13: Regime HMs, 60 months rolling estimation window for means and variances, λ=2, 5, and 10

This table shows the results for regimes higher moments (HMs) following Platanakis, Sakkas and Sutcliffe (2019) for λ=2, 5, and 10. We consider a 60 months rolling estimation window with monthly rebalancing. CTA: Commodity Trading Advisors; ED: Event Driven; GM: Global Macro; LO: Long Only; LS: Long Short; MN: Market Neutral; MS: Multi Strategy; RV: Relative Value; SE: Sector; SB: Short Bias; OT: Others. We examine the statistical significance of the SR difference between the E+B+rf and the SR of the 11 portfolios which also include a HF strategy. * denotes significance at p < 0.1, ** denotes significance at p < 0.05 and *** denotes significance at p < 0.01.

		Benchmark Portfolio (E+B+rf) plus each individual HF Style											
Portfolio Construction	**Performance Metric**	**E+B+rf**	**(+CTA)**	**(+ED)**	**(+GM)**	**(+LO)**	**(+LS)**	**(+MN)**	**(+MS)**	**(+RV)**	**(+SE)**	**(+SB)**	**(+OT)**
Regimes HMs λ=2	Sharpe Ratio	0.0266	0.7752***	0.3621**	0.3690**	0.3048*	0.6452***	-0.0268	0.4140**	0.2061	0.4567***	0.0251	0.8733***
	MPPM (Θ%)	-0.9252	6.5005	2.3458	2.5336	2.0542	4.6849	-1.0922	2.7149	0.9987	3.3568	-0.7785	5.2601
	Omega Ratio	1.1884	2.1027	1.5819	1.5915	1.4661	1.9257	1.1699	1.7411	1.4595	1.6590	1.1826	2.4674
	Sterling Ratio	0.0014	0.2024	0.0323	0.0414	0.0247	0.1685	-0.0014	0.0356	0.0156	0.0558	0.0016	0.3440
λ=5	Sharpe Ratio	0.0720	0.8808***	0.4901**	0.5512***	0.2969	0.6896***	0.1583	0.5301**	0.4326***	0.3858*	0.1277	1.1120***
	MPPM (Θ%)	-0.1342	6.5301	3.0795	3.4993	1.8491	4.4874	0.6148	3.2419	2.5783	2.5728	0.3878	5.5777
	Omega Ratio	1.2795	2.2965	1.7862	1.9117	1.5005	2.0613	1.4322	1.9921	1.8177	1.6258	1.3477	3.0491
	Sterling Ratio	0.0046	0.3026	0.0530	0.1072	0.0231	0.1749	0.0167	0.0580	0.0485	0.0404	0.0122	0.4851
λ=10	Sharpe Ratio	0.2051	0.8440***	0.7152**	0.7335***	0.3389	0.5817**	0.3580	0.8000***	0.7445***	0.3617	0.1942	1.2435***
	MPPM (Θ%)	0.8938	5.3150	4.0240	3.8375	1.9506	3.2083	1.5902	4.0976	3.6505	2.1473	0.7944	5.5162
	Omega Ratio	1.5083	2.3085	2.1988	2.3125	1.6377	2.0191	1.8301	2.5020	2.4627	1.6918	1.5450	3.6002
	Sterling Ratio	0.0209	0.3251	0.0815	0.2743	0.0282	0.1297	0.0689	0.1461	0.1097	0.0357	0.0228	0.5449